

A reproducible UQ framework for ocean heat content via local conditional simulations

Thea Sukianto¹, Mikael Kuusela¹, Donata Giglio²

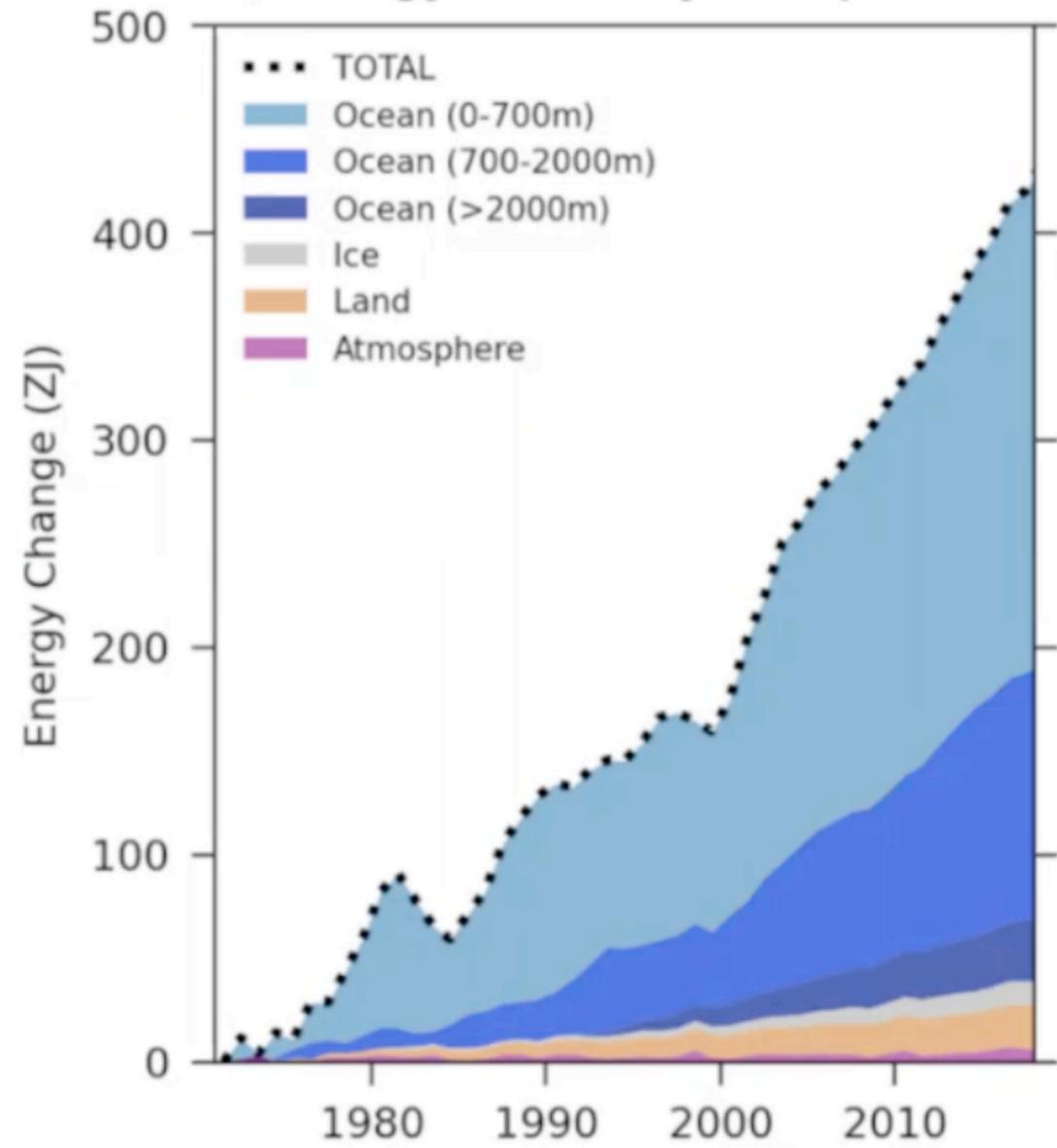
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JSM 2024

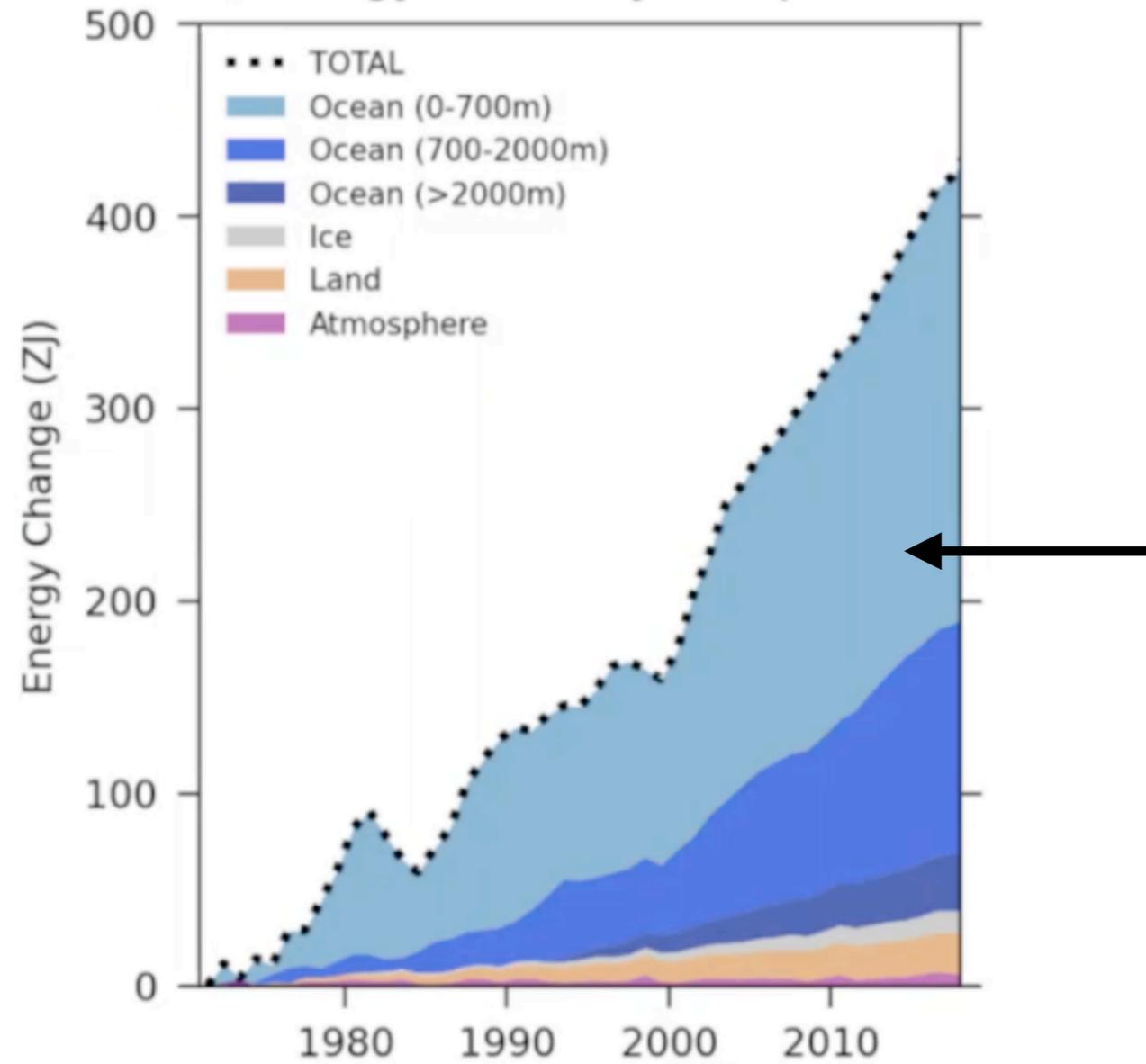
4 August 2024

Changes in OHC contribute to extreme climate events



(IPCC AR6)

Changes in OHC contribute to extreme climate events



~**91%** of the excess heat in the climate system is stored **in the ocean.**

(IPCC AR6)

Changes in OHC contribute to extreme climate events

Is sea level rising?

Yes, sea level is rising at an increasing rate



With continued ocean and atmospheric warming, sea levels will likely rise for many centuries at rates higher than in the United States, almost 40 percent of the population lives in relatively high-population-density coastal areas. Globally, eight of the world's 10 largest cities are coastal. U.N. Atlas of the Oceans.

(NOAA NOS, 2024)

Climate change is probably increasing the intensity of tropical cyclones

BY , MAYA V. CHUNG, GABE VECCHI, JINGRU SUN, , AND
PUBLISHED MARCH 31, 2021

HIGHLIGHTS

- Warming of the surface ocean from human-induced climate change is likely fueling more powerful tropical cyclones (TCs).
- The destructive power of individual TCs through flooding is amplified by rising sea level, which very likely has a substantial contribution at the global scale from anthropogenic climate change.
- TC precipitation rates are projected to increase due to enhanced atmospheric moisture associated with anthropogenic global warming.
- The proportion of Category 4 & 5 TCs has increased, possibly due to anthropogenic climate change, and is

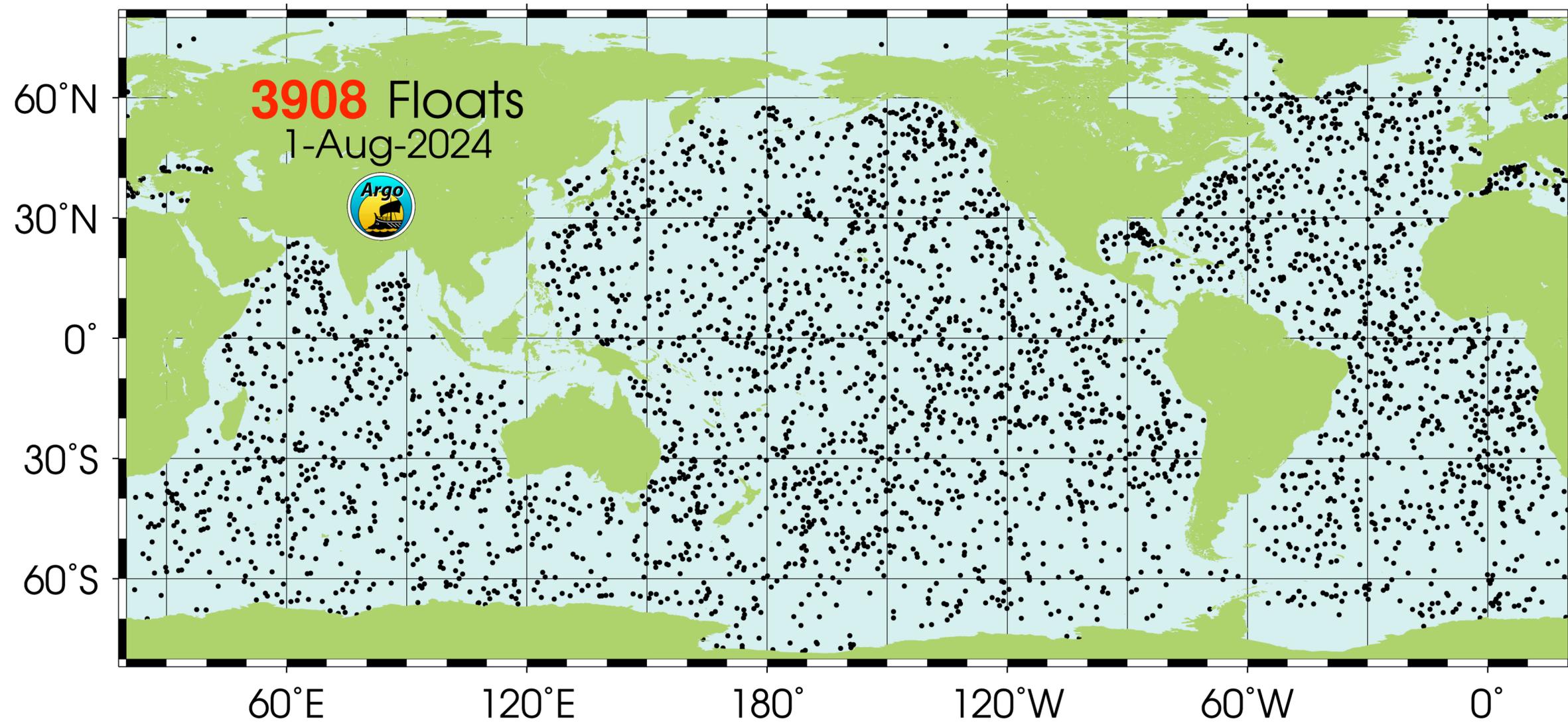
This post is adapted from a [ScienceBrief](#) by the same authors. The findings are consistent with the Intergovernmental Panel on Climate Change's [Special Report on the Oceans and Cryosphere in a Changing Climate](#) and with the reports ([Part 1](#), [Part 2](#)) from the World Meteorological Organization's Task Team on Tropical Cyclones and Climate Change. The images that appear in the post were not part of the original brief.

This [ScienceBrief](#) presents a summary of the state of the science on tropical cyclones (tropical storms, hurricanes, and typhoons) and climate change. The authors assessed more than 90 peer-reviewed scientific articles, with a focus on articles describing observations of, or projected future changes to, the frequency and intensity of tropical cyclones (TCs) globally or in key regions, as well as changes in tropical cyclone-related rainfall and storm surge.

Warming of the surface ocean from anthropogenic (human-induced) climate change is likely fueling more powerful TCs. The destructive power of individual TCs through flooding is amplified by rising sea level, which very likely has a substantial contribution at the global scale from anthropogenic climate change. In addition, TC precipitation rates are projected to increase due to enhanced atmospheric moisture associated with anthropogenic global warming.

(NOAA Climate.gov, 2021)

We use Argo data (2004-2022) to estimate 15-1850 dbar OHC

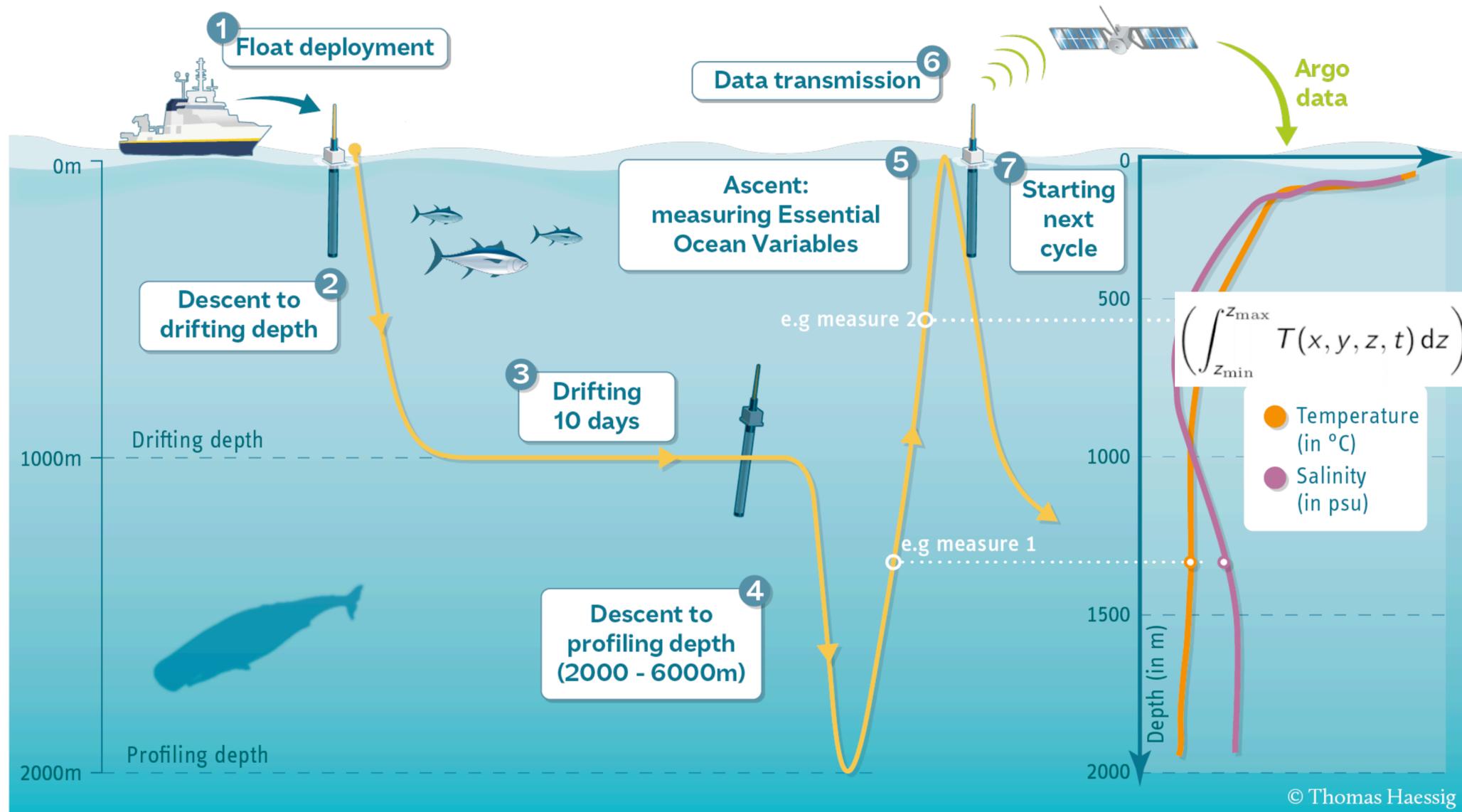


(Argo Program)



We use Argo data (2004-2022) to estimate 15-1850 dbar OHC

$$\text{OHC}(t) = \rho_0 c_{p,0} \iint \left(\int_{z_{\min}}^{z_{\max}} T(x, y, z, t) dz \right) dx dy$$

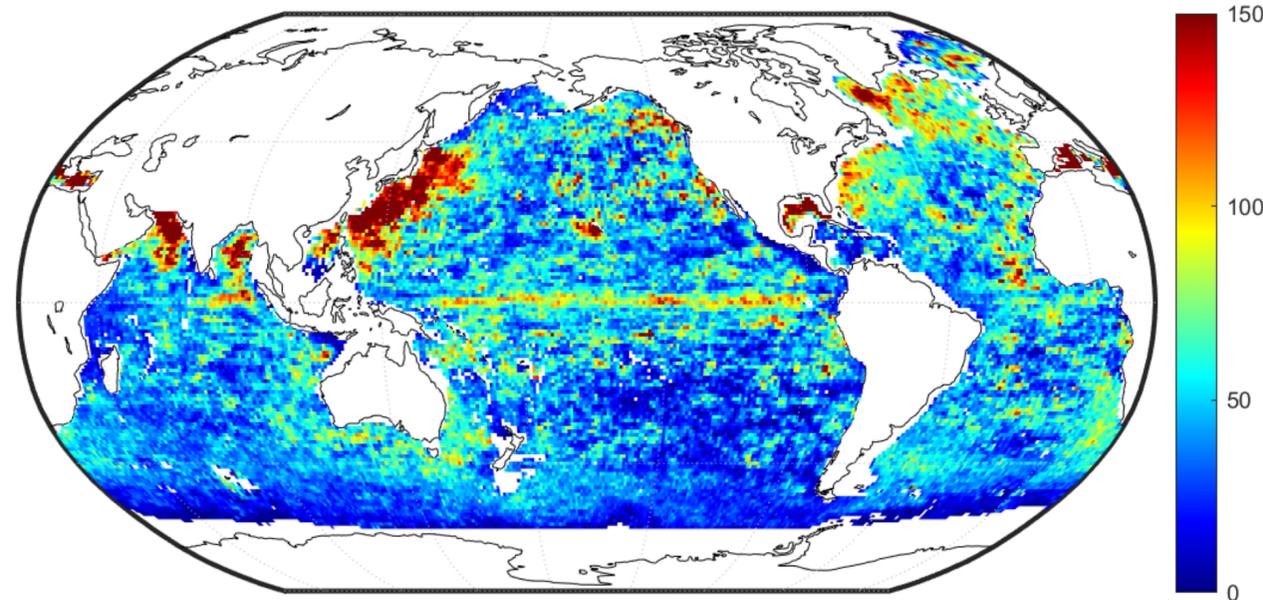


(Argo Program)

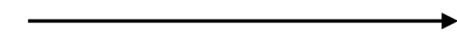
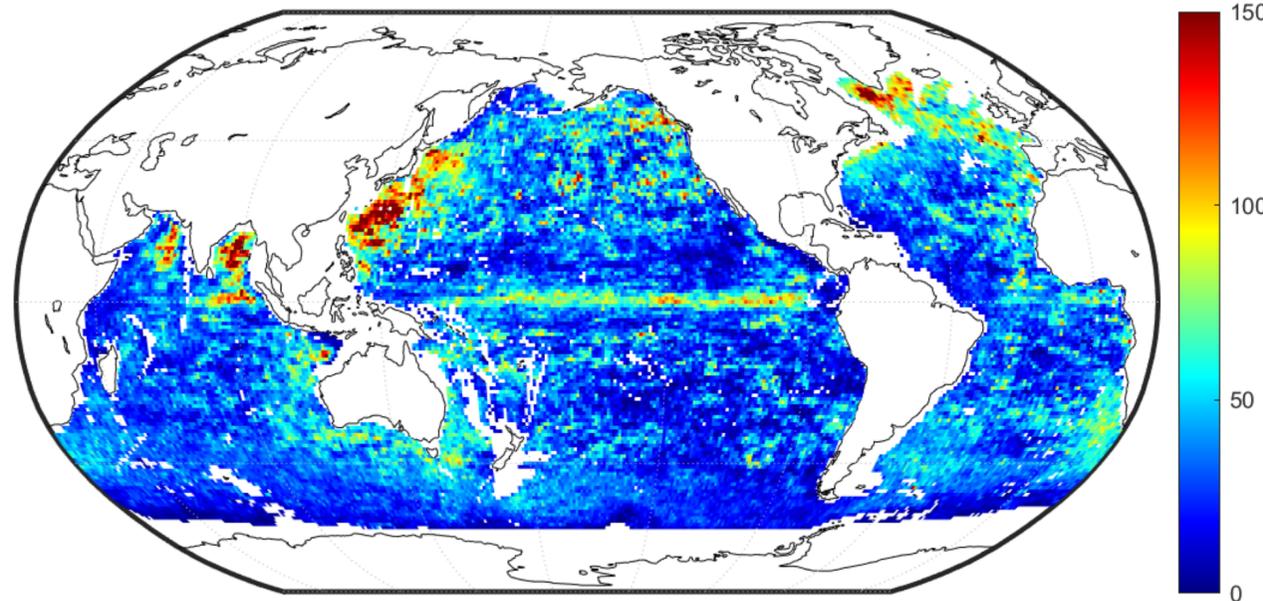
Modeling OHC from sparse observations: a 4-step framework

1. Integrate the **vertical dimension**
2. Model **horizontal and temporal dimension**
3. Estimate **uncertainties** based on (1) and (2)
4. Integrate the **horizontal dimension**

We use Argo data (2004-2022) to estimate 15-1850 dbar OHC



Top layer profiles (15-975 dbar)



Bottom layer profiles (975-1850 dbar)

Modeling OHC from sparse observations: a 4-step framework

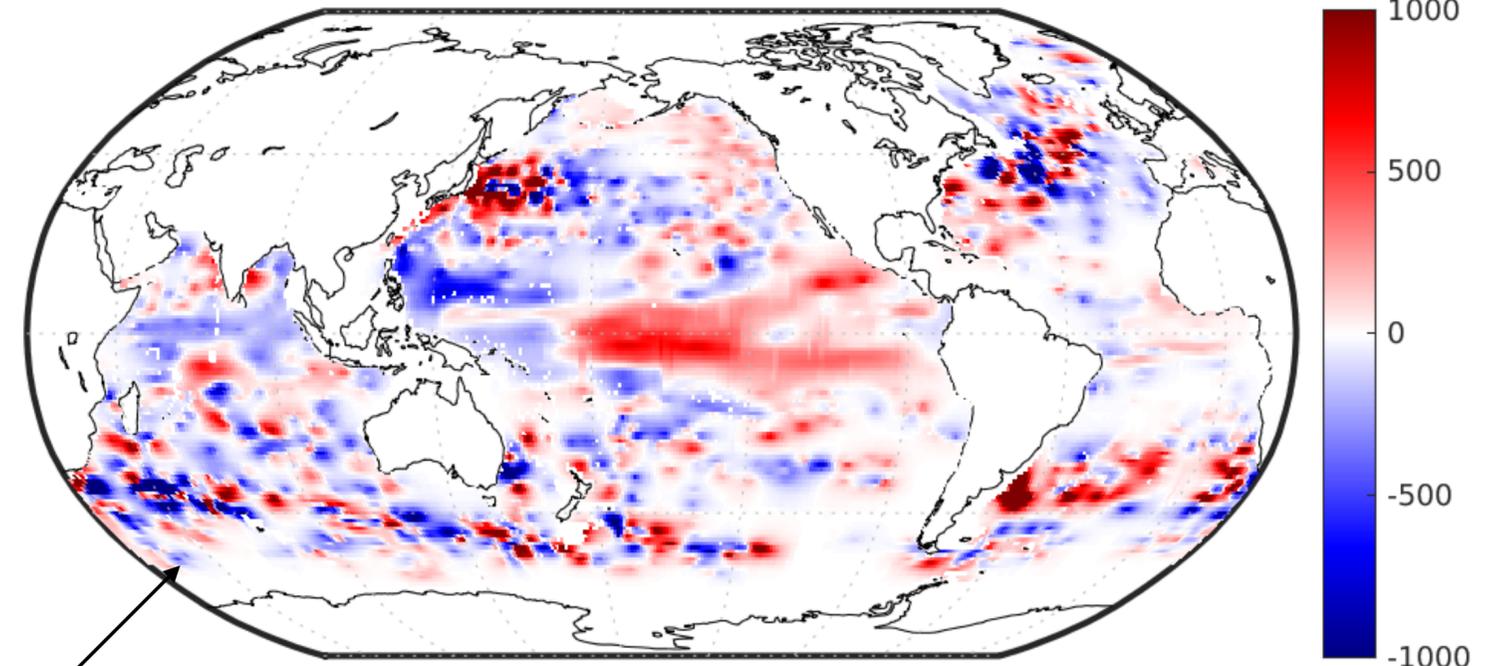
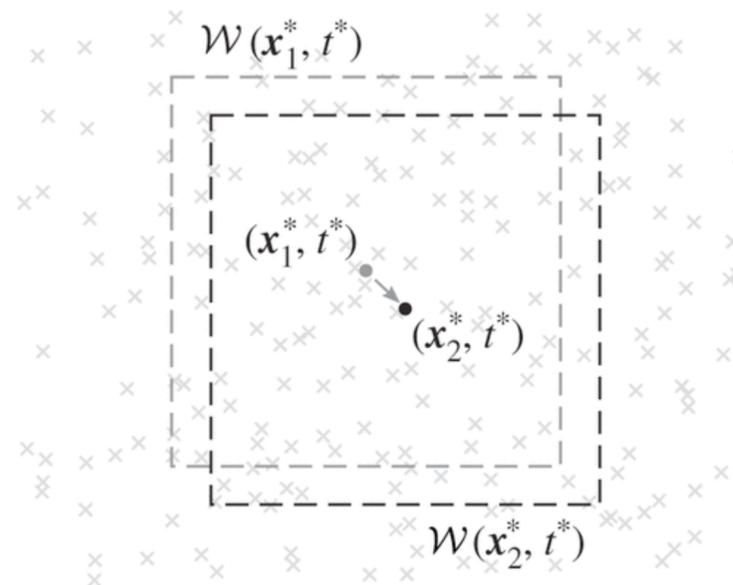
1. Integrate the **vertical dimension**
2. Model **horizontal and temporal dimension**
3. Estimate **uncertainties** based on (1) and (2)
4. Integrate the **horizontal dimension**

Modeling challenges arise from the data size and nonstationarity

- **Size:** > 2.5 million Argo profiles (matrix inversion for covariance parameter estimation and kriging is infeasible)
- **Nonstationarity:** Challenging to define a nonstationary covariance function flexible enough to explain variability across entire global ocean

We can estimate a gridded temperature field with GP regression

1. Model temperature mean field: local least squares regression (Roemmich and Gilson 2009) + linear time trend
2. Model residuals: **locally stationary** Gaussian process (GP) regression (Kuusela and Stein 2018)



Modeled top layer residuals (02/2005)

We can estimate a gridded temperature field with GP regression

$$\begin{bmatrix} y_{\text{top}} \\ y_{\text{bot}} \end{bmatrix}_{i,j} = f_i \left(\begin{bmatrix} x_{\text{top}} \\ x_{\text{bot}} \end{bmatrix}_{i,j}, \begin{bmatrix} t_{\text{top}} \\ t_{\text{bot}} \end{bmatrix}_{i,j} \right) + \begin{bmatrix} \epsilon_{\text{top}} \\ \epsilon_{\text{bot}} \end{bmatrix}_{i,j}$$

Temperature
residuals

Latitude
Longitude

Date

Nugget
effect

We can estimate a gridded temperature field with GP regression

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Temperature residuals

Latitude
Longitude

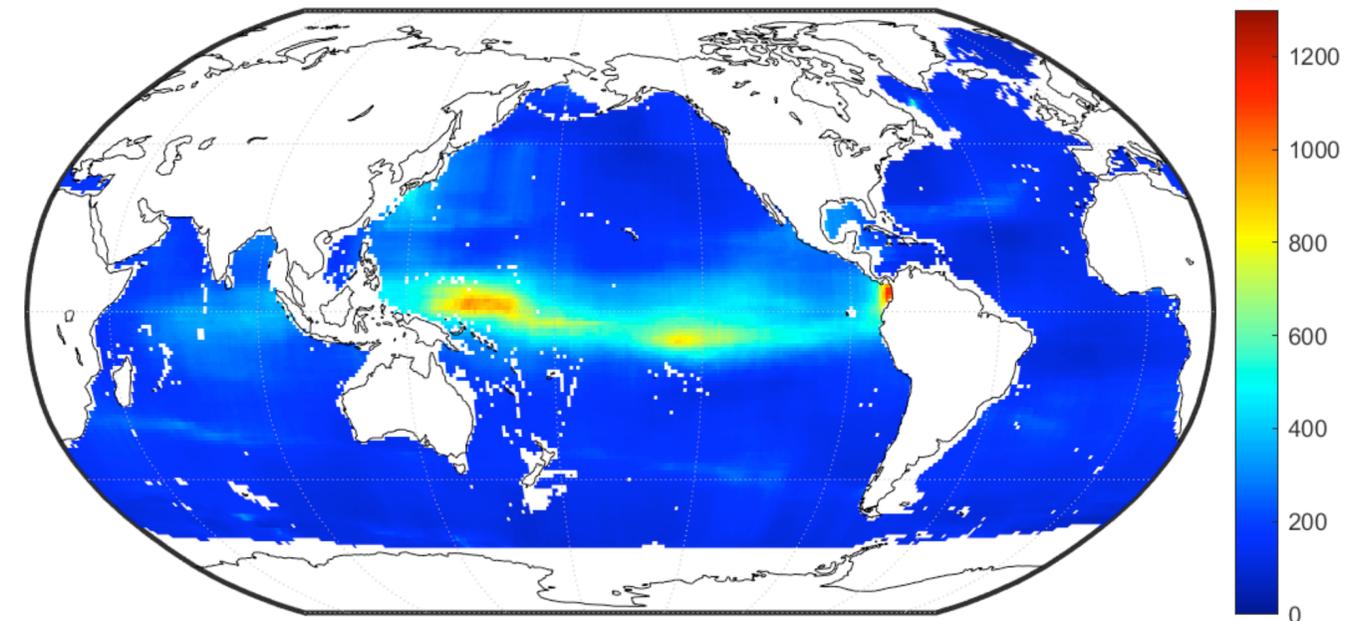
Date

Nugget effect

$$f_i \stackrel{\text{iid}}{\sim} \text{GP} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{K}(x_1, t_1, x_2, t_2; \boldsymbol{\theta}) \right)$$

$$\begin{bmatrix} \epsilon_{\text{top}} \\ \epsilon_{\text{bot}} \end{bmatrix} \stackrel{\text{iid}}{\sim} \text{N} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma}_{\epsilon}(\boldsymbol{\theta}_{\epsilon}) \right)$$

$$\mathbf{K}_{ii}(z_1, z_2; \boldsymbol{\theta}) = \frac{\delta_i^2}{\sqrt{|\boldsymbol{\Theta}_i|}} \exp \left(-\sqrt{(z_1 - z_2)^{\text{T}} \boldsymbol{\Theta}_i^{-1} (z_1 - z_2)} \right)$$



Length scale MLE (latitude; top layer)

Modeling OHC from sparse observations: a 4-step framework

1. Integrate the vertical dimension
2. Model **horizontal and temporal dimension**
3. Estimate **uncertainties** based on (1) and (2)
4. Integrate the **horizontal dimension**

Obtaining uncertainties is facilitated by local conditional simulations

full field | data ?

Obtaining uncertainties is facilitated by local conditional simulations

full field | data - **multivariate normal** with conditional covariance Σ_i

(parameterized by estimated GP variance, length scales, cross-correlation)

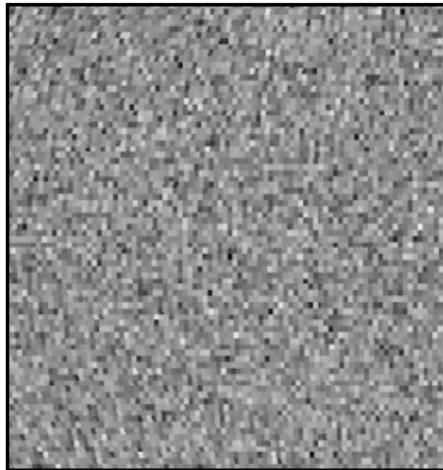
Obtaining uncertainties is facilitated by local conditional simulations

full field | data - **multivariate normal** with conditional covariance Σ_i

(parameterized by estimated GP variance, length scales, cross-correlation)

Local conditional simulations! (extension of Nychka et.al. 2018)

Obtaining uncertainties is facilitated by local conditional simulations

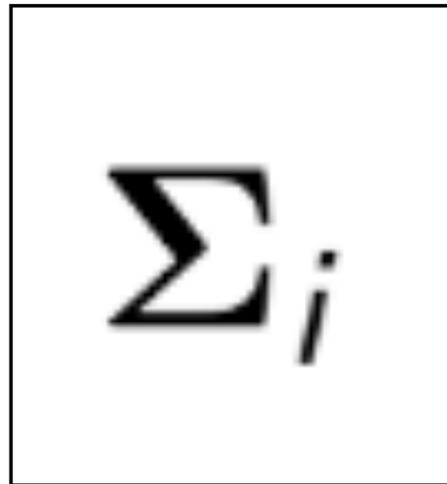
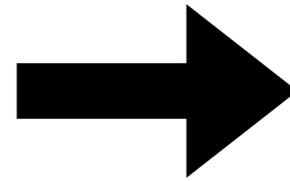


Simulate Gaussian
white noise over grid
(keep fixed)

Obtaining uncertainties is facilitated by local conditional simulations

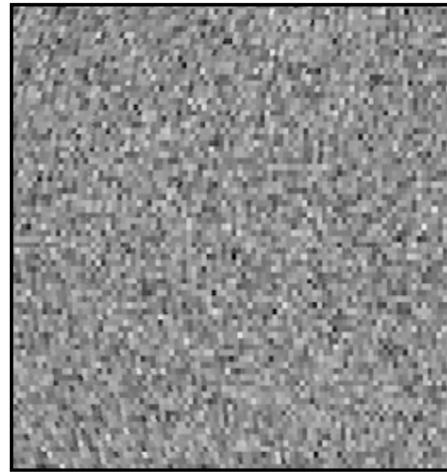


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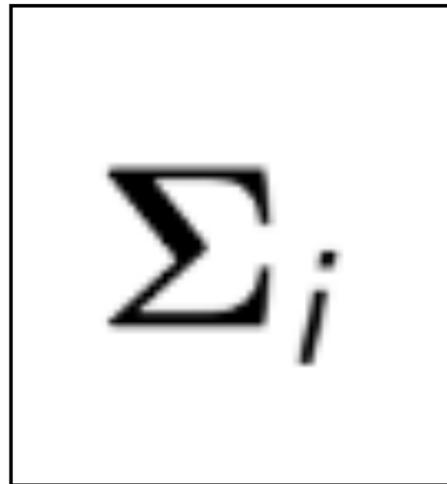
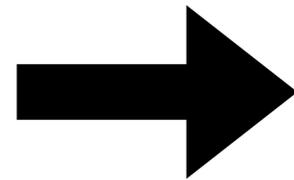


Compute (local)
conditional covariance
matrix

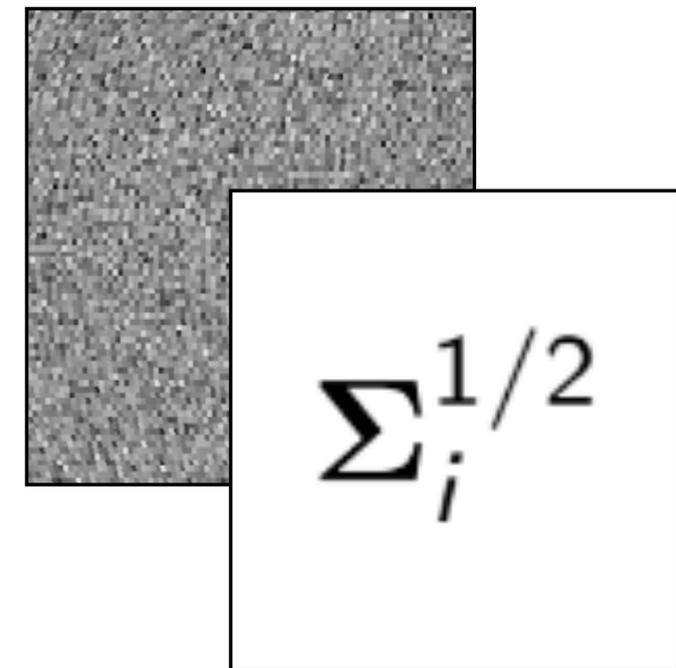
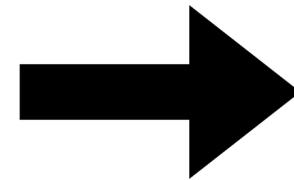
Obtaining uncertainties is facilitated by local conditional simulations



Simulate Gaussian white noise over grid (keep fixed)

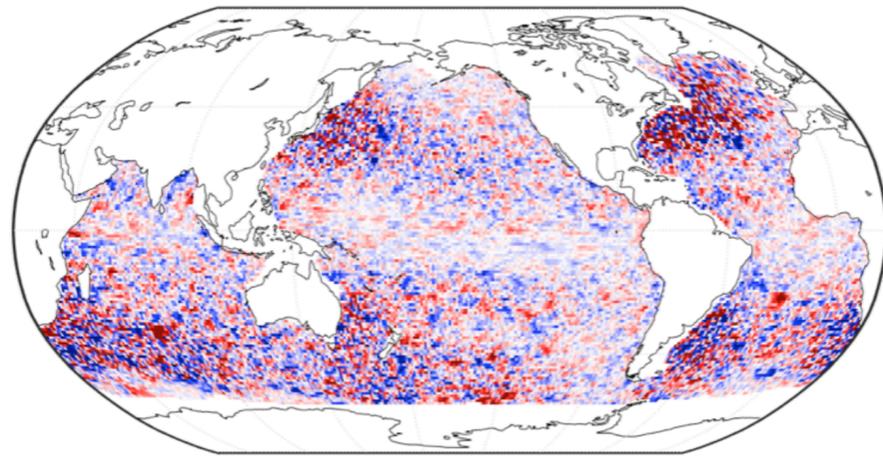


Compute (local) conditional covariance matrix



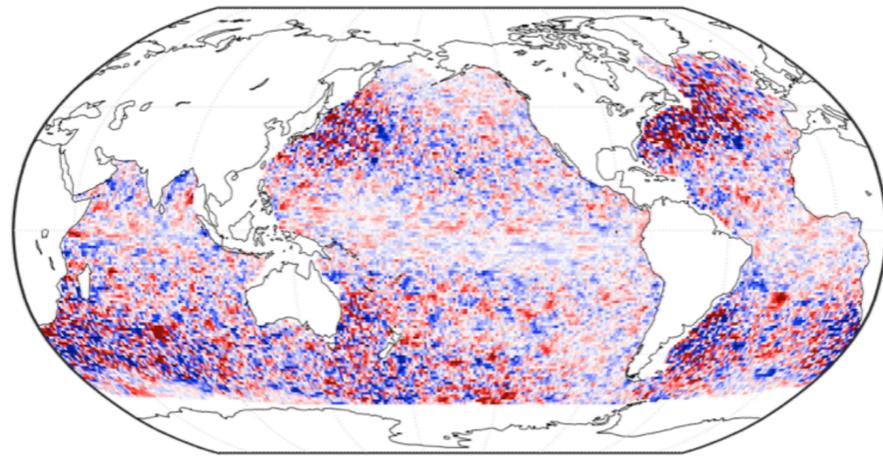
Take symmetric square root and multiply by white noise

Obtaining uncertainties is facilitated by local conditional simulations

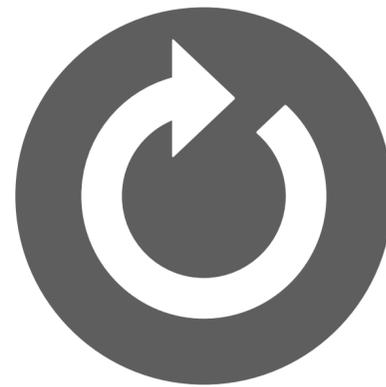
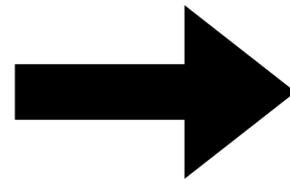


Keep the center point
and repeat for all grid
points

Obtaining uncertainties is facilitated by local conditional simulations

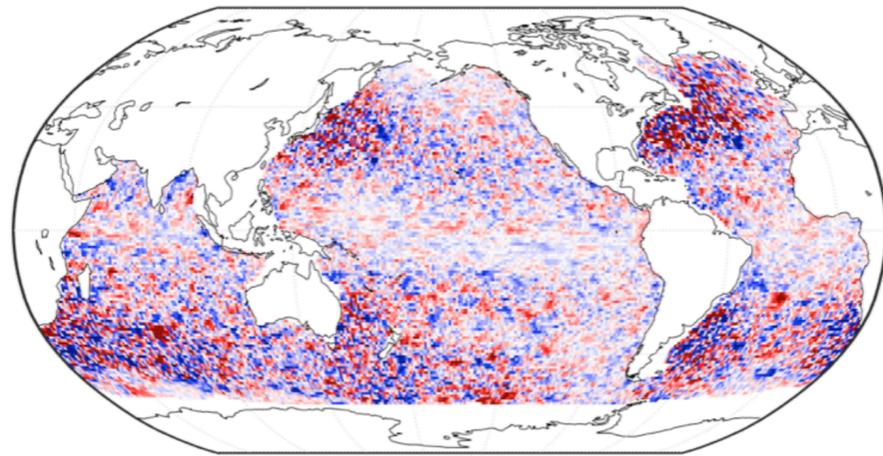


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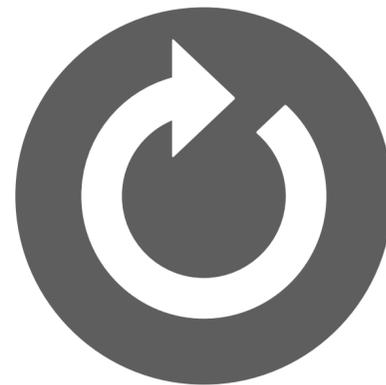
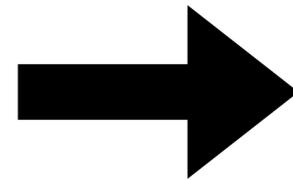


Repeat for desired
number of ensemble
members and other
layer

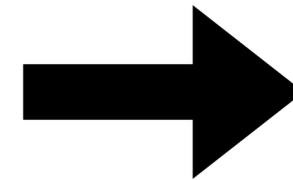
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Keep the center point
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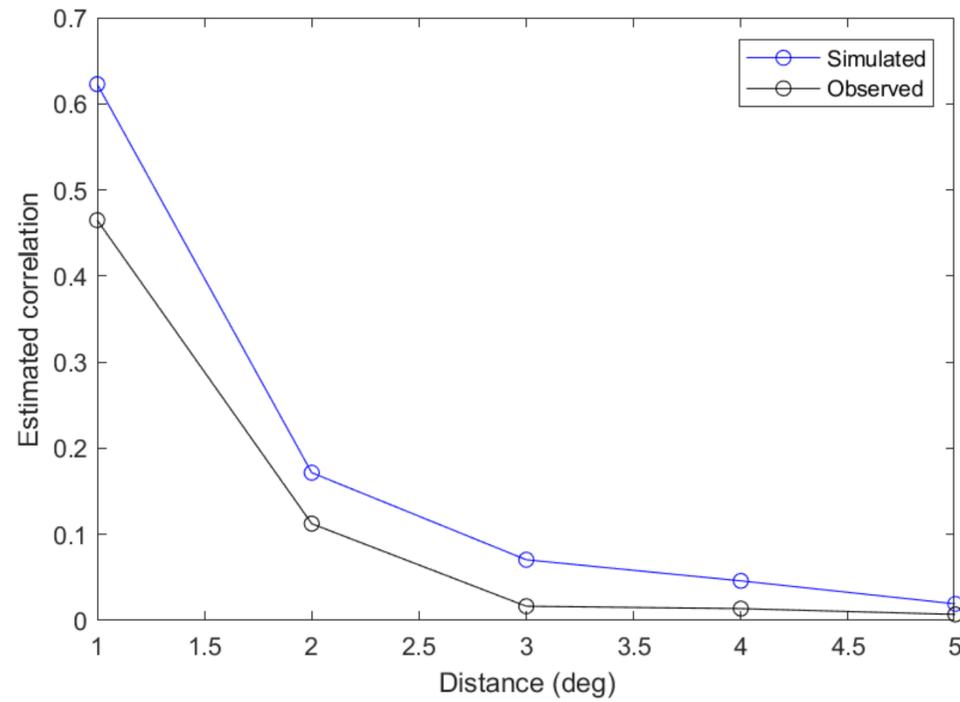
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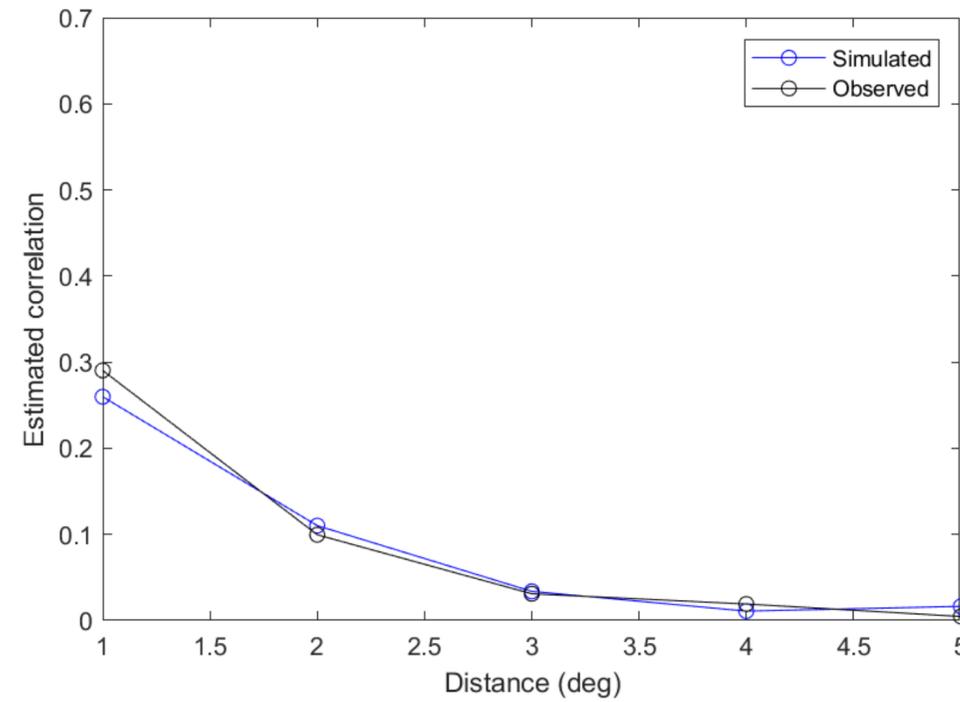
Compute conservative estimate of
OHC total conditional variance
using sample variances

$$(\sqrt{\text{Var}(\text{OHC}_{\text{top}}|\text{data})} + \sqrt{\text{Var}(\text{OHC}_{\text{bot}}|\text{data})})^2$$

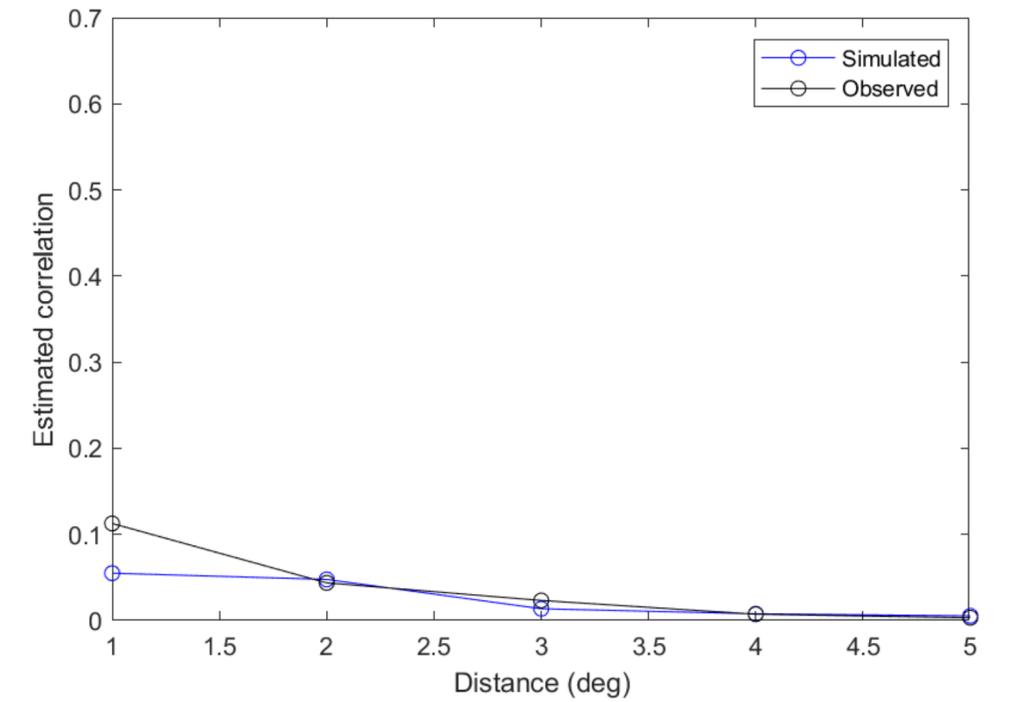
CV shows simulations capture conditional spatial dependence reasonably well



0~30 days



30~60 days

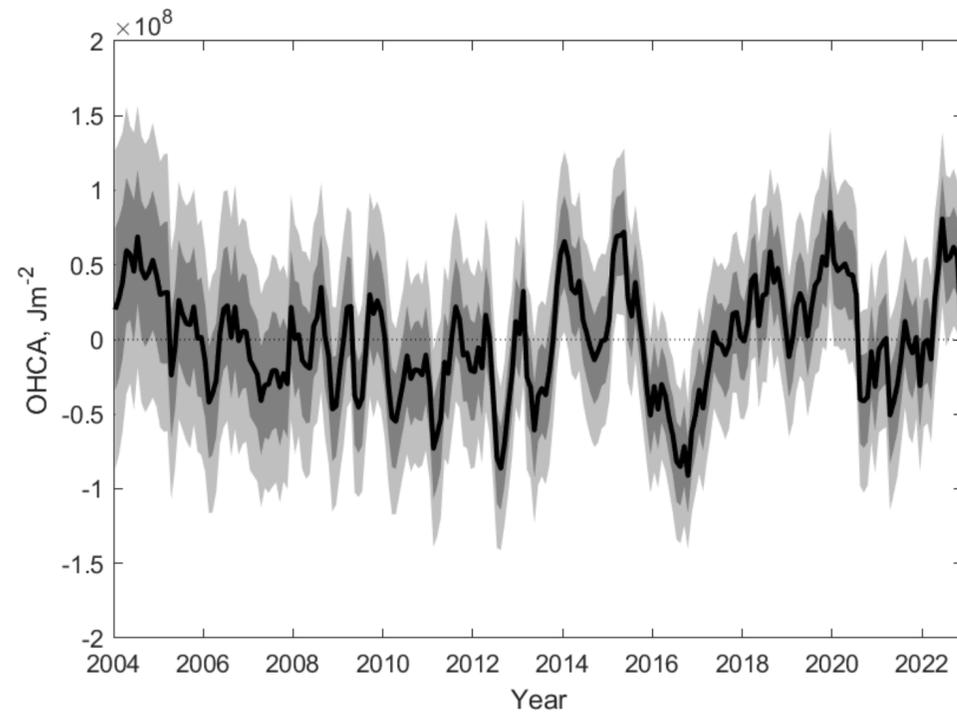


60~90 days

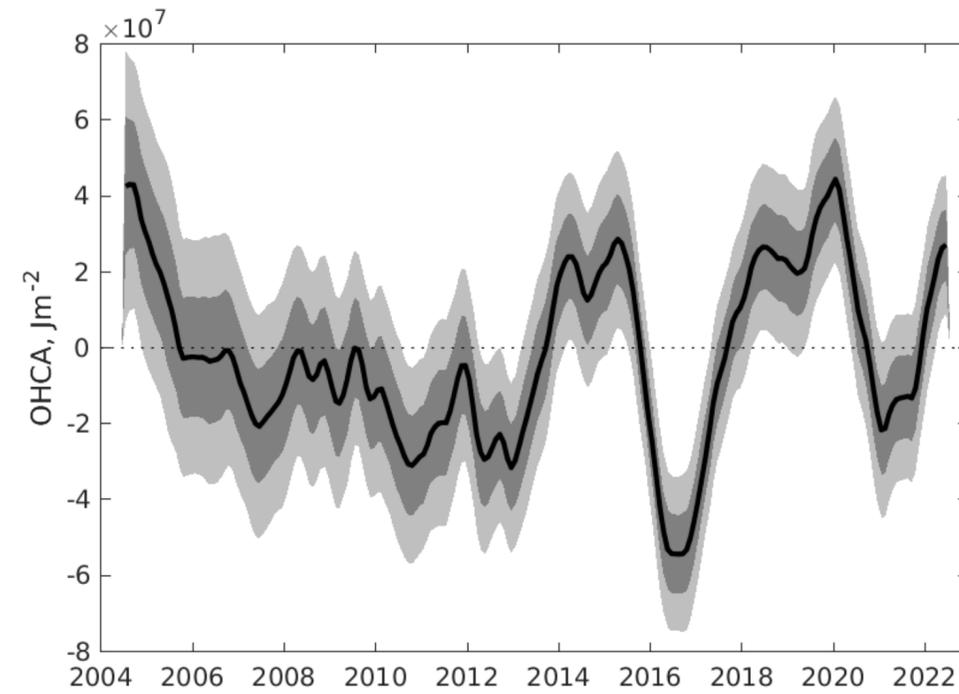
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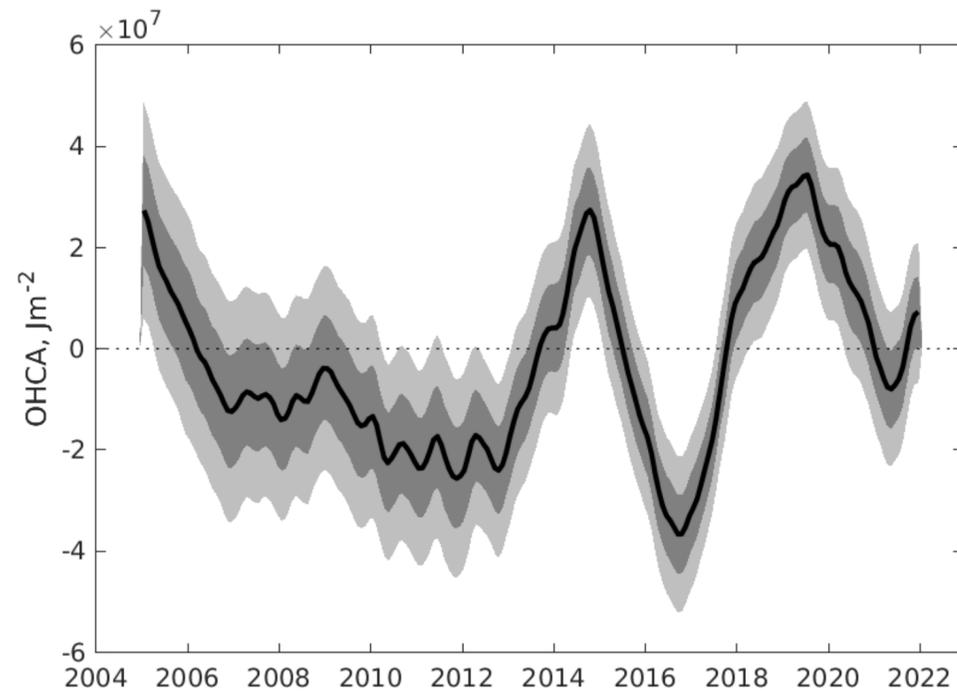
Uncertainties: Global OHC anomalies



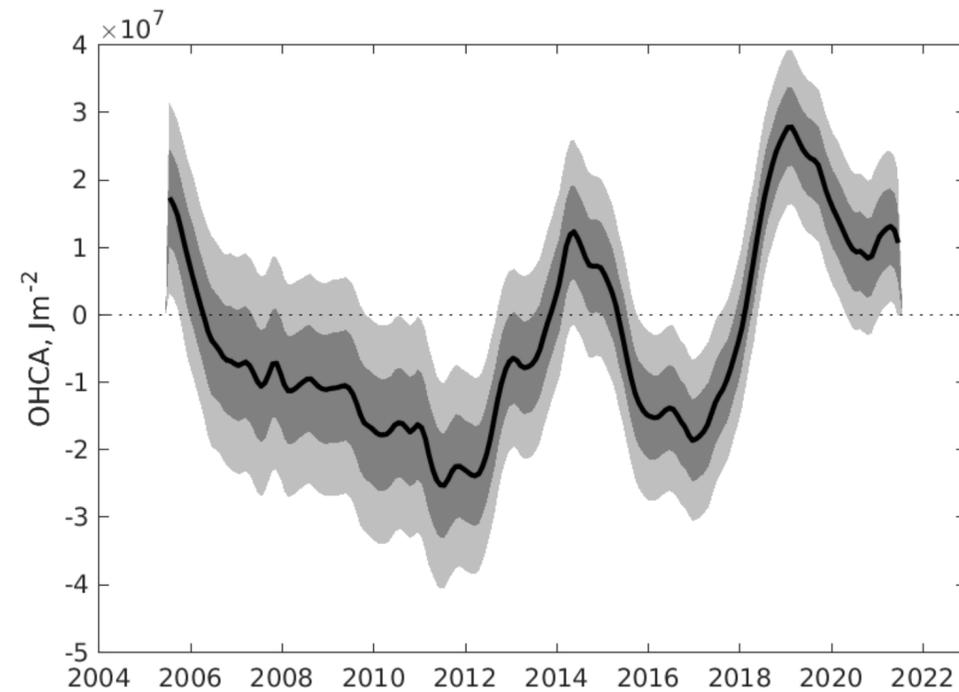
Monthly



12-month
moving average

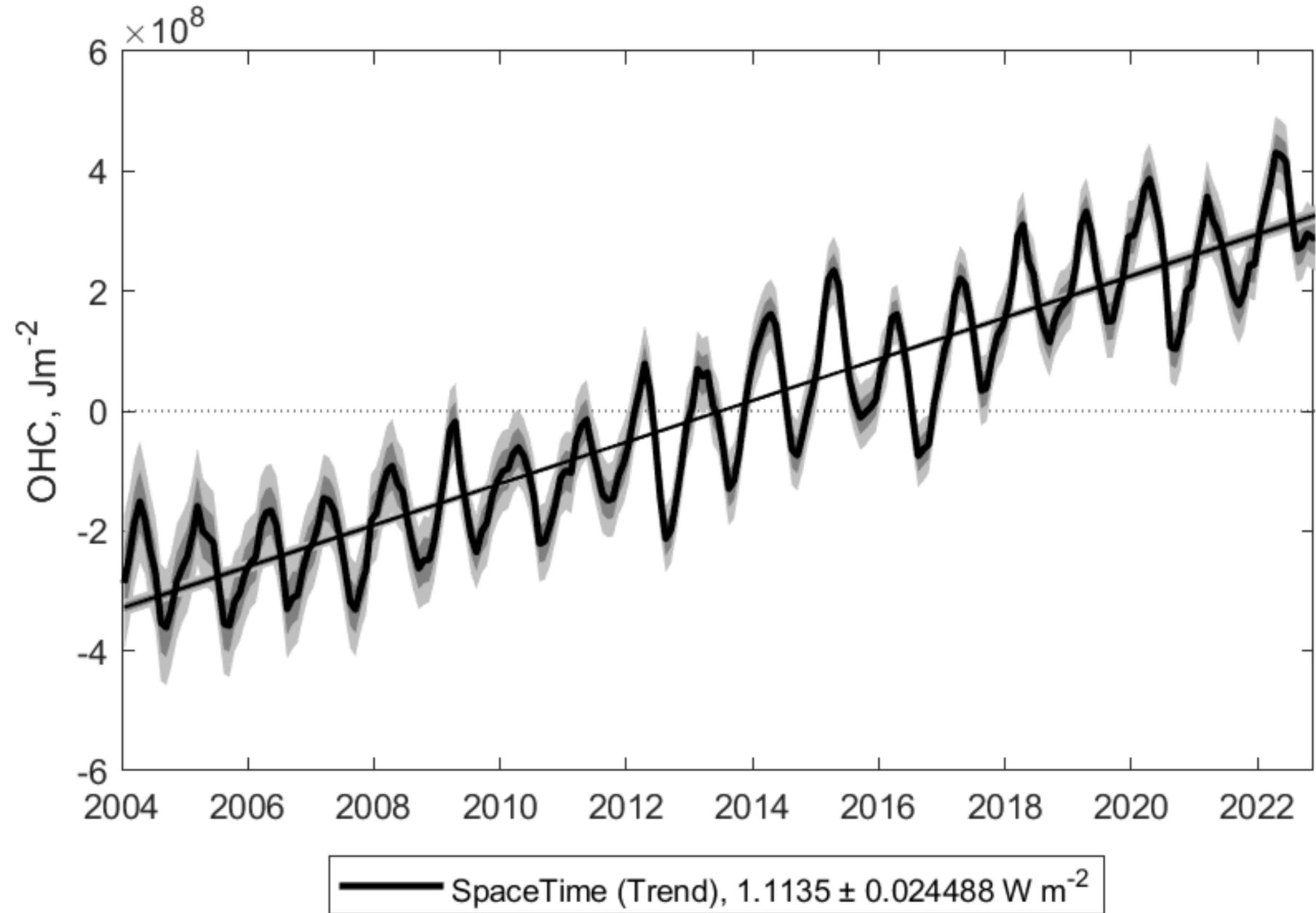


24-month
moving average

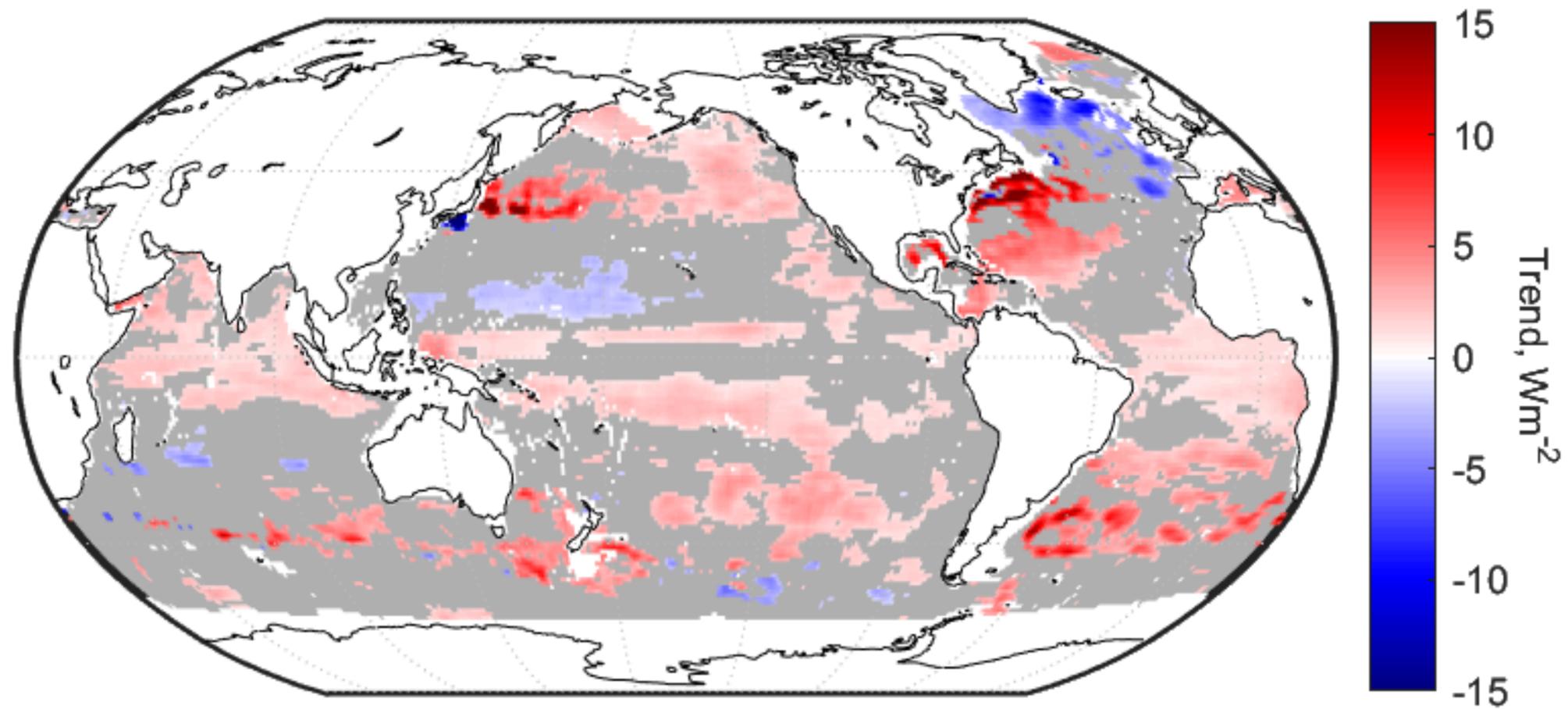


36-month
moving average

Uncertainties: Global OHC trend

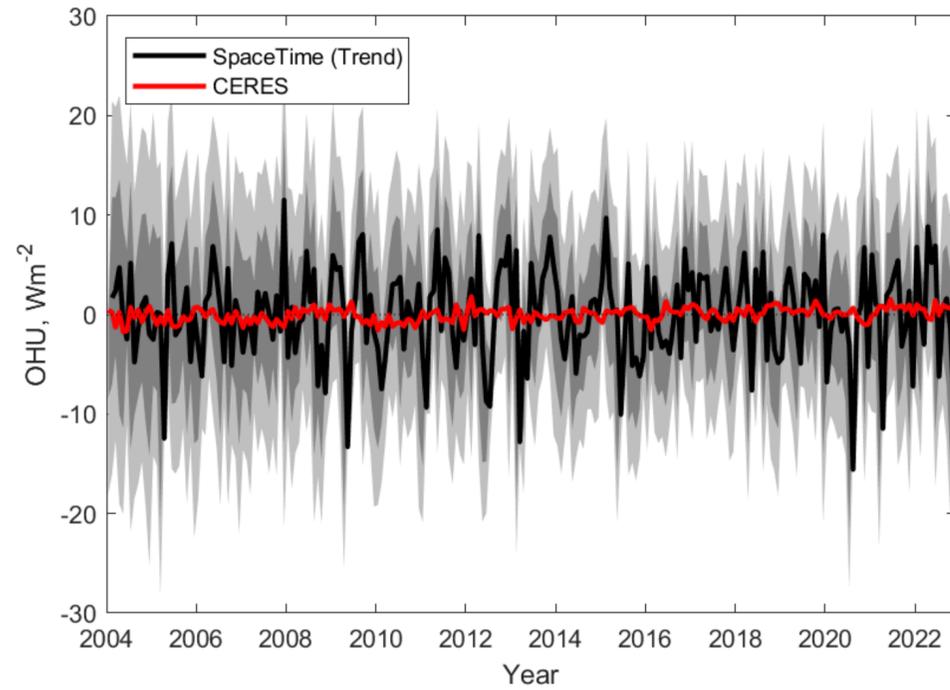


Uncertainties: Regional OHC trends

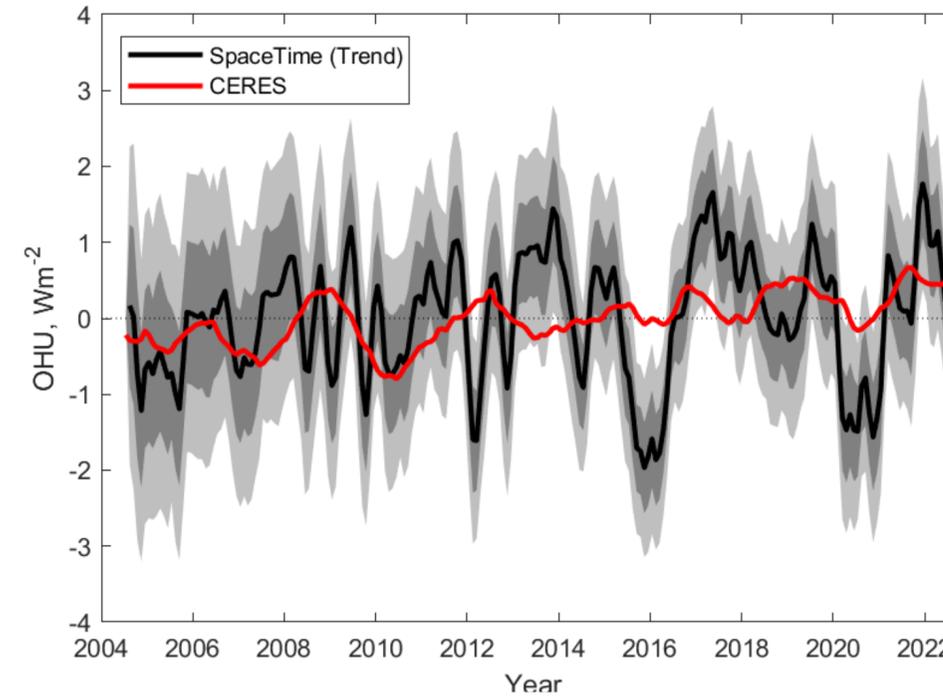


Significant OHC trends at 5% level (2004-2022)

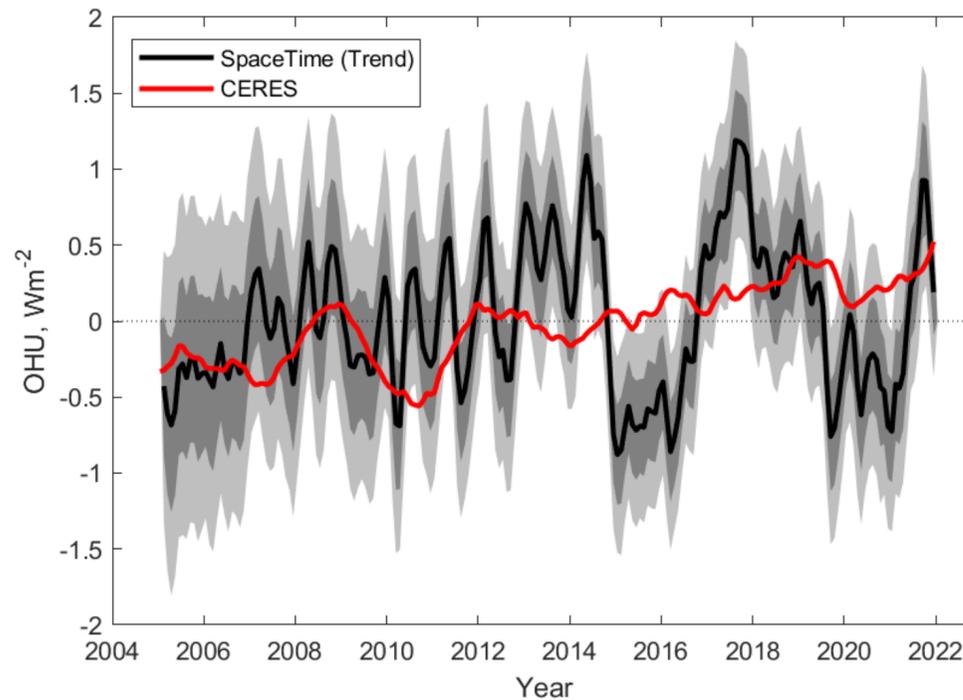
Uncertainties: Global ocean heat uptake (OHU) anomalies



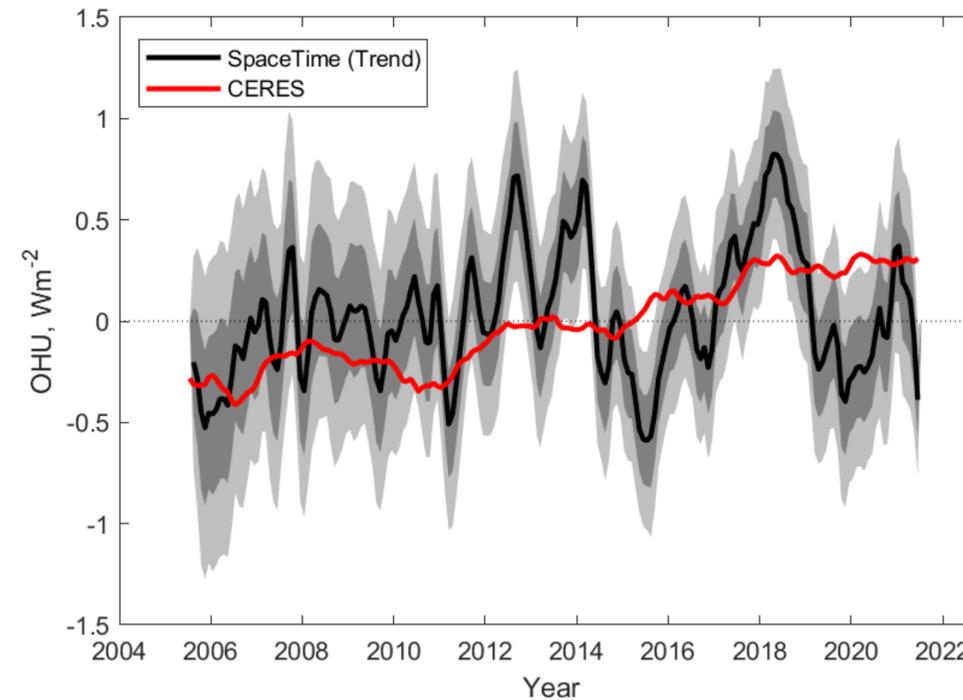
Monthly



12-month
moving average

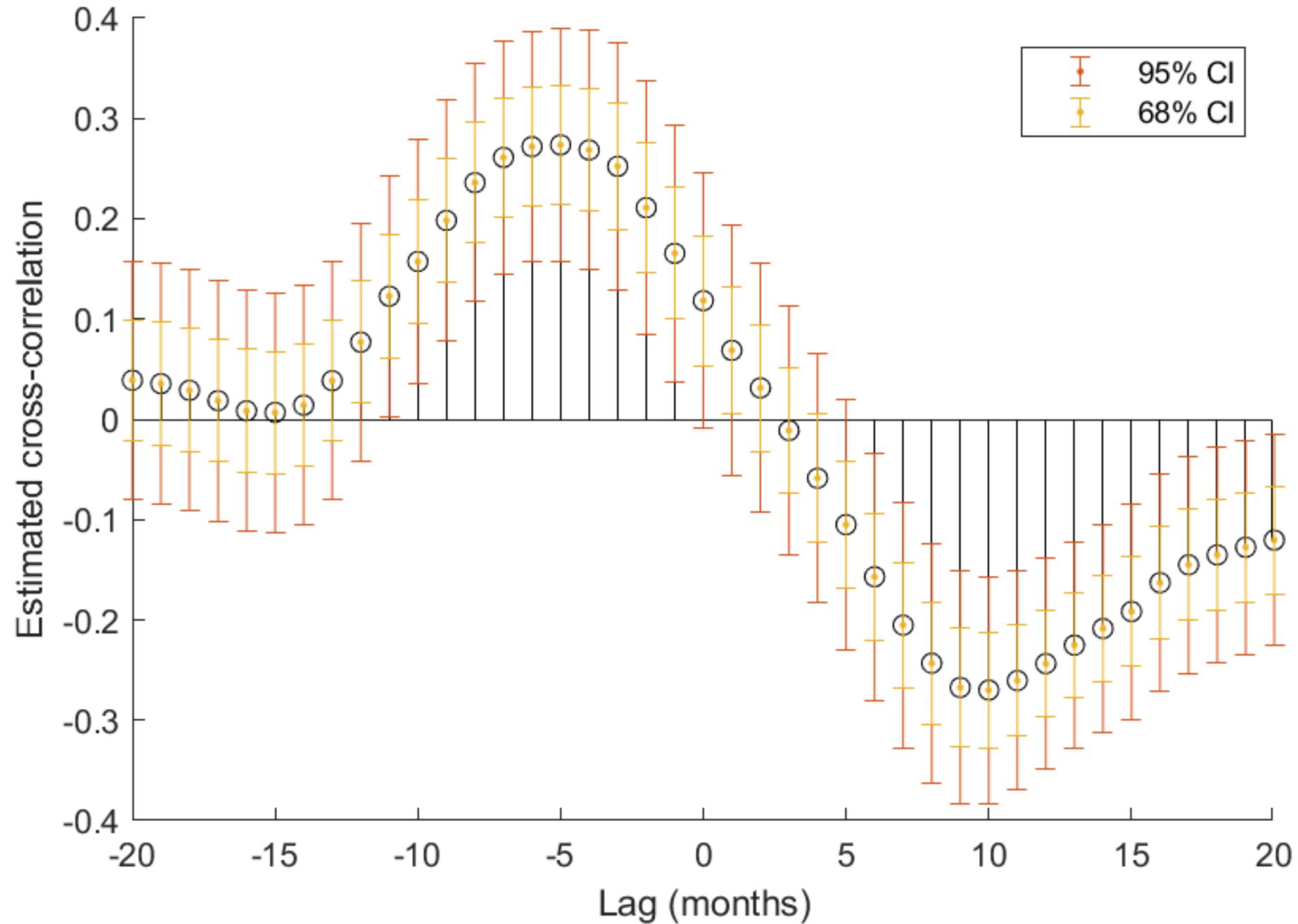


24-month
moving average

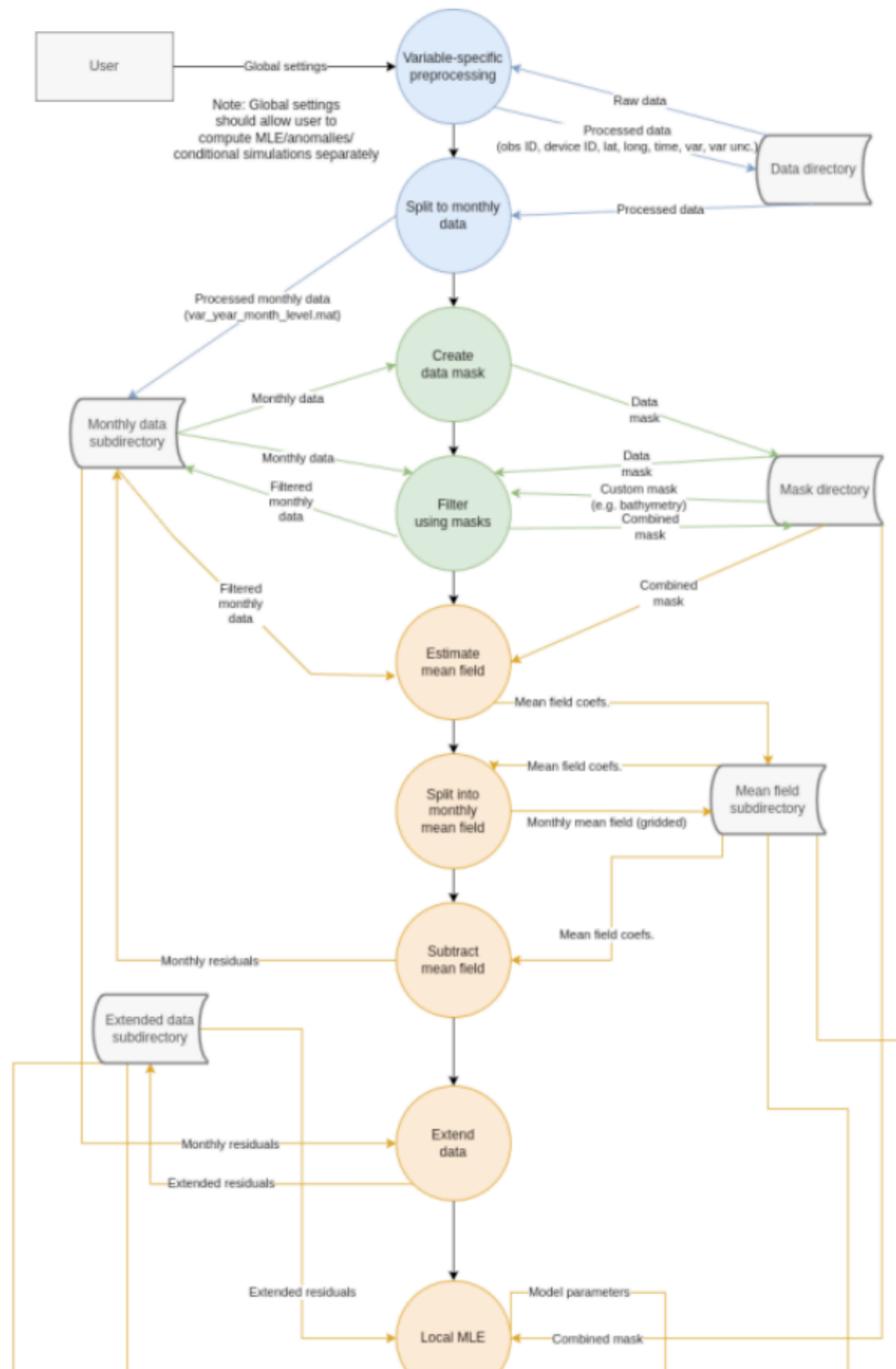


36-month
moving average

Uncertainties: Correlation between OHC anomalies and ONI



A reproducible, open-source pipeline (work-in-progress)



The screenshot shows the GitHub repository page for **OHC_analysis**, forked from **mkuusela/OHC_analysis**. The repository is **Private** and has **3 Branches** and **0 Tags**. The current branch is **code-only**. The repository is **50 commits ahead of, 1 commit behind** **mkuusela/OHC_analysis:master**. The repository has **517 Commits** and was last updated **2 months ago** by **ttsukianto**. The repository contains the following files and folders:

- Data**: Remove all results from code-only branch (last year)
- cbrewer**: update to plots after Mikael's email on 7 May 2018 on... (6 years ago)
- .gitattributes**: Back up 2021 results (last year)

The framework in action: Data product

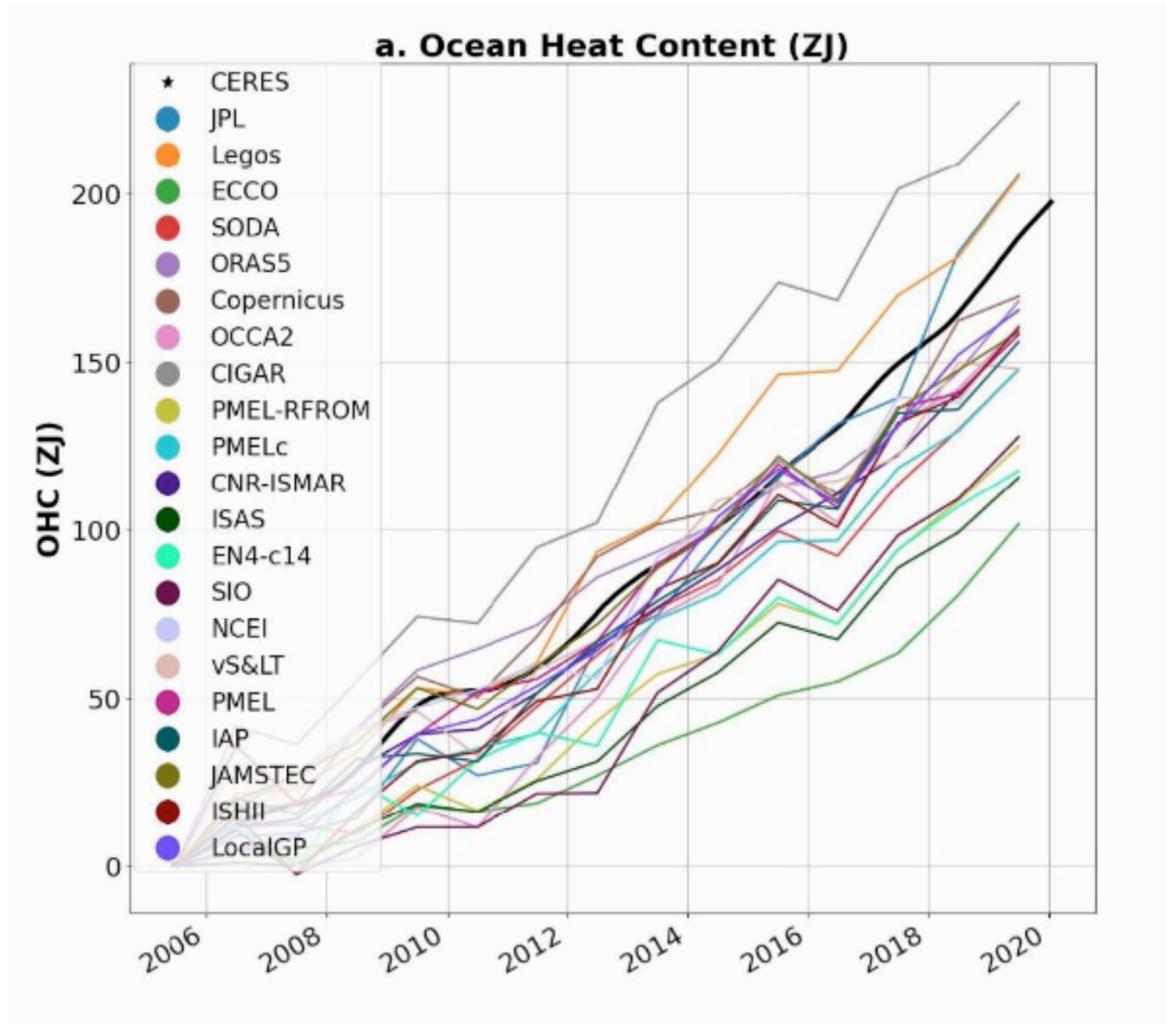
Global Ocean Heat Content Anomalies and Ocean Heat Uptake based on mapping Argo data using local Gaussian processes

Giglio, Donata¹ ; Sukianto, Thea²; Kuusela, Mikael²

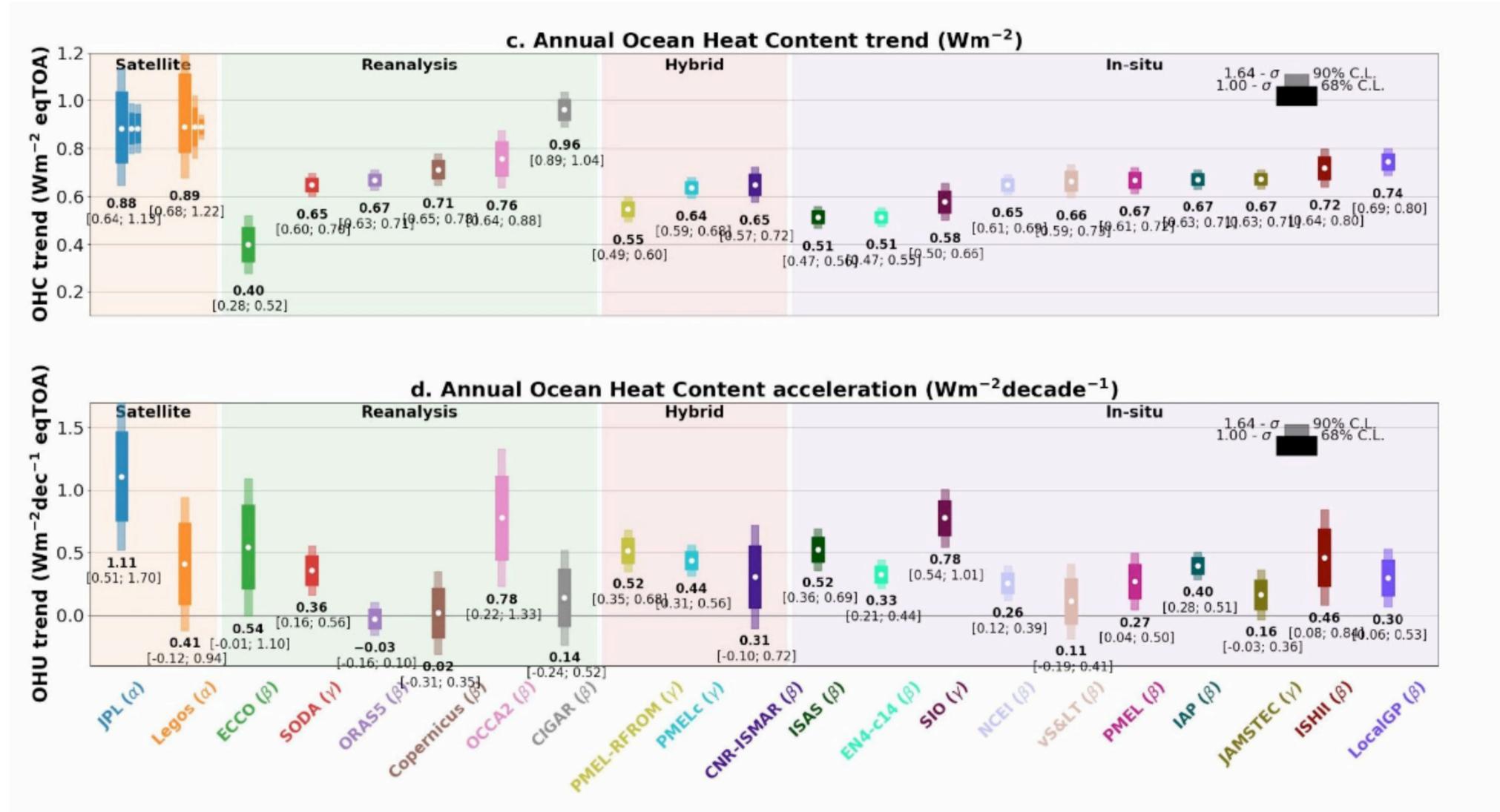
Show affiliations

NOTE for users: please use the latest version of the product. Monthly Ocean Heat Content Anomalies (OHCA) in the top 2000 dbar of the ocean are calculated (during 2004-2022, equatorward of 65 degree latitude) subtracting the mean over the period 2004-2022 from the monthly time series of OHC. Yearly OHCA time series are then calculated that include two points per year, i.e., from averaging Jan to Dec and Jul to Jun, respectively. OHC fields are mapped using locally stationary Gaussian processes (defined over space and time) with data-driven decorrelation scales (Kuusela and Stein, 2018). A linear time trend was included in the estimate of the mean field (along with spatial terms and harmonics for the annual cycle). Mapping is done separately for different vertical sections: 15-20 dbar, 15-300 dbar, 300-700 dbar, 700-1850 dbar, 1800-1850 dbar. The 15-20 dbar (1800-1850 dbar) section is used to estimate OHCA for 0-15 dbar (1850-2000 dbar), where observations are sparser. Different vertical sections are combined to estimate global OHCA time series for 0-2000 dbar. The attribute "area" is included in the netcdf files and it tells the corresponding surface area for the estimates. Regions of the ocean that are shallower than 300 m or are not sufficiently well sampled by the Argo array are not included. Ocean Heat Uptake is also calculated from the monthly OHCA and then averaged as described above to produce a yearly time series and included in the file.

The framework in action: Intercomparison efforts



(Hakuba et. al 2024)



A previous version of this data product was contributed to the 2022 WMO State of the Global Climate Report.

The implementation is still computationally challenging

- For every grid point, use data in a 10 degree/3 month window
- Estimate GP model parameters numerically with MLE + BFGS algorithm
- How many grid points? **360 long x 180 lat = 64,800 grid points (!)**
- Embarrassingly parallel, but still computationally challenging
 - Fit parameters (optimize all): **~48h @ PSC**
 - Obtain conditional simulations for Feb of every year: **~48h**

Conclusions

- Estimating ocean heat content (with reliable uncertainties) is crucial for **tracking climate change and mitigation**
- To address nonstationarity and the large Argo dataset size, we model the temperature residuals with **locally stationary Gaussian process regression**
- To estimate the uncertainties of the total OHC in the water column (top + bottom), we generate ensembles of **local conditional simulations**
- Using this reproducible framework, we are able to obtain reliable uncertainties for **OHCA, regional/global OHC trends, and OHU**

Future work:

Bivariate extension, SSH/OHC joint mapping, neural inference (Walchessen et al. 2024)...

Questions? Comments? Concerns?

Contact: thea@stat.cmu.edu

Data product

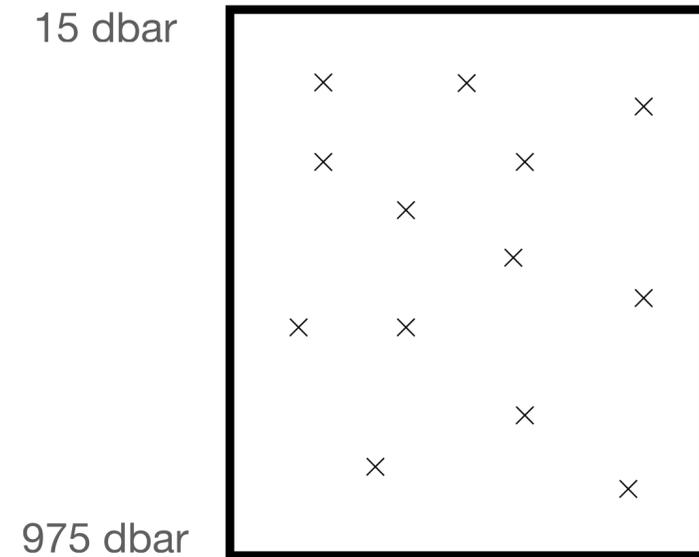


Intercomparison paper



Backup

How do we estimate uncertainties for the total (top + bottom layers) OHC?



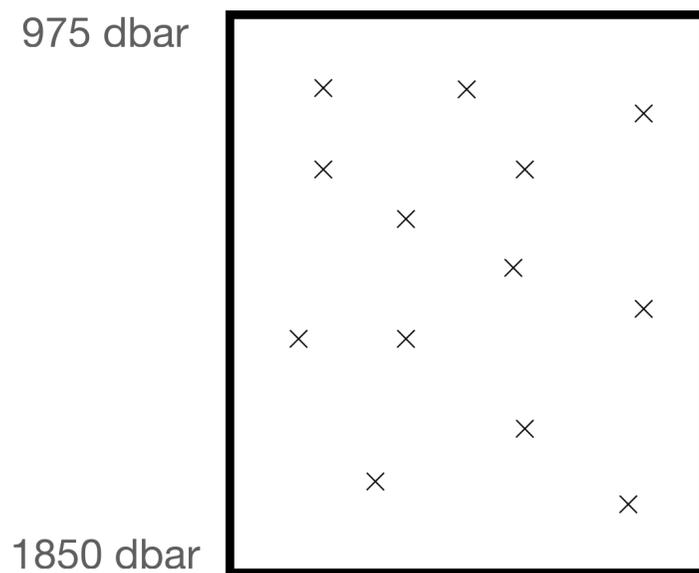
$$\text{Var}(\text{OHC}_{\text{top}}|\text{data})$$



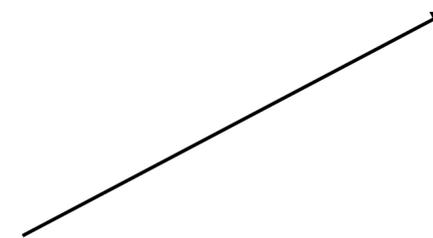
$$\text{Var}(\text{OHC}_{\text{total}}|\text{data})$$


A large red 'X' is drawn over the total variance equation, indicating that this method is incorrect.

$$\text{Var}(\text{OHC}_{\text{top}}|\text{data}) + \text{Var}(\text{OHC}_{\text{bot}}|\text{data})$$

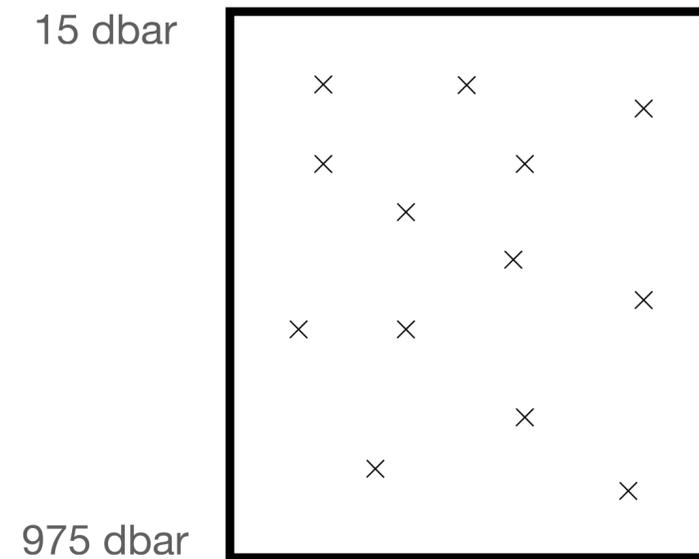


$$\text{Var}(\text{OHC}_{\text{bot}}|\text{data})$$

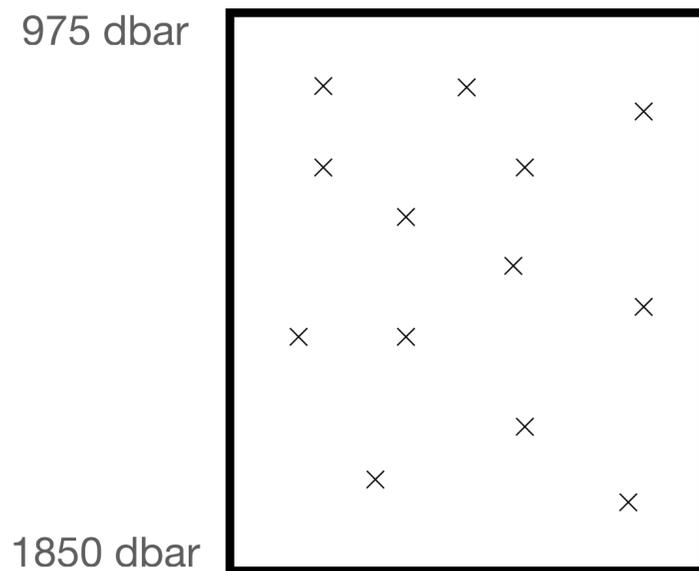


Summing the variances for each layer **underestimates** the uncertainties of the total OHC.

How do we estimate uncertainties for the total (top + bottom layers) OHC?

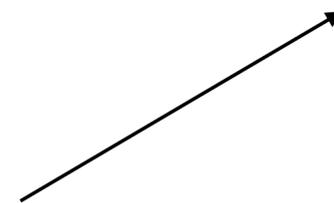


$$2\text{Cov}(\text{OHC}_{\text{top}}, \text{OHC}_{\text{bot}} | \text{data})$$



$$\text{Var}(\text{OHC}_{\text{top}} | \text{data})$$

$$\text{Var}(\text{OHC}_{\text{bot}} | \text{data})$$



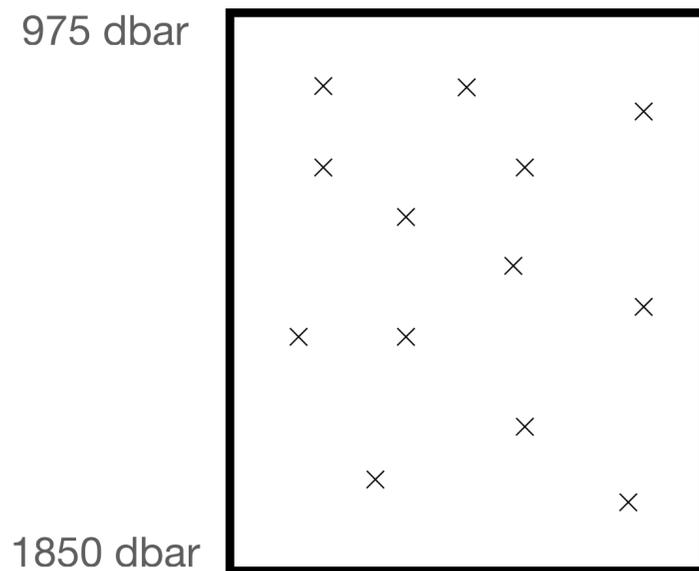
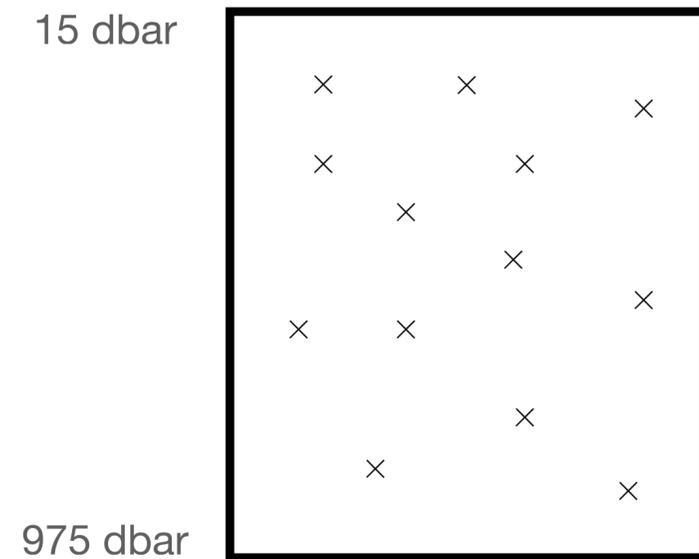
$$\text{Var}(\text{OHC}_{\text{total}} | \text{data})$$

(conservative upper bound)

$$(\sqrt{\text{Var}(\text{OHC}_{\text{top}} | \text{data})} + \sqrt{\text{Var}(\text{OHC}_{\text{bot}} | \text{data})})^2$$

Squaring the sum of the standard deviations for each layer **overestimates** the uncertainties of the total OHC.

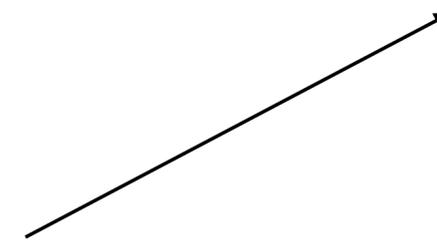
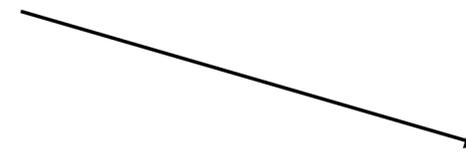
How do we estimate uncertainties for the total (top + bottom layers) OHC?



$$\text{Var}(\text{OHC}_{\text{top}}|\text{data})$$

$$2\text{Cov}(\text{OHC}_{\text{top}}, \text{OHC}_{\text{bot}}|\text{data})$$

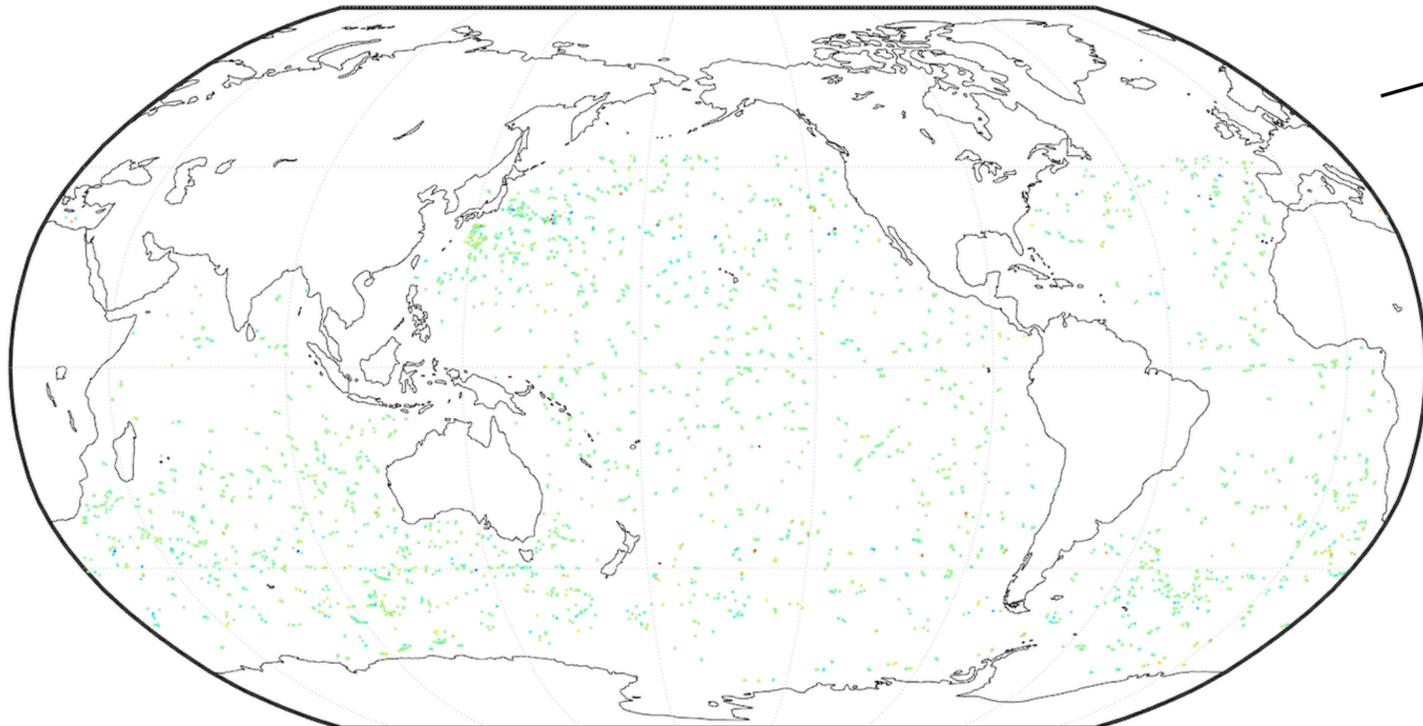
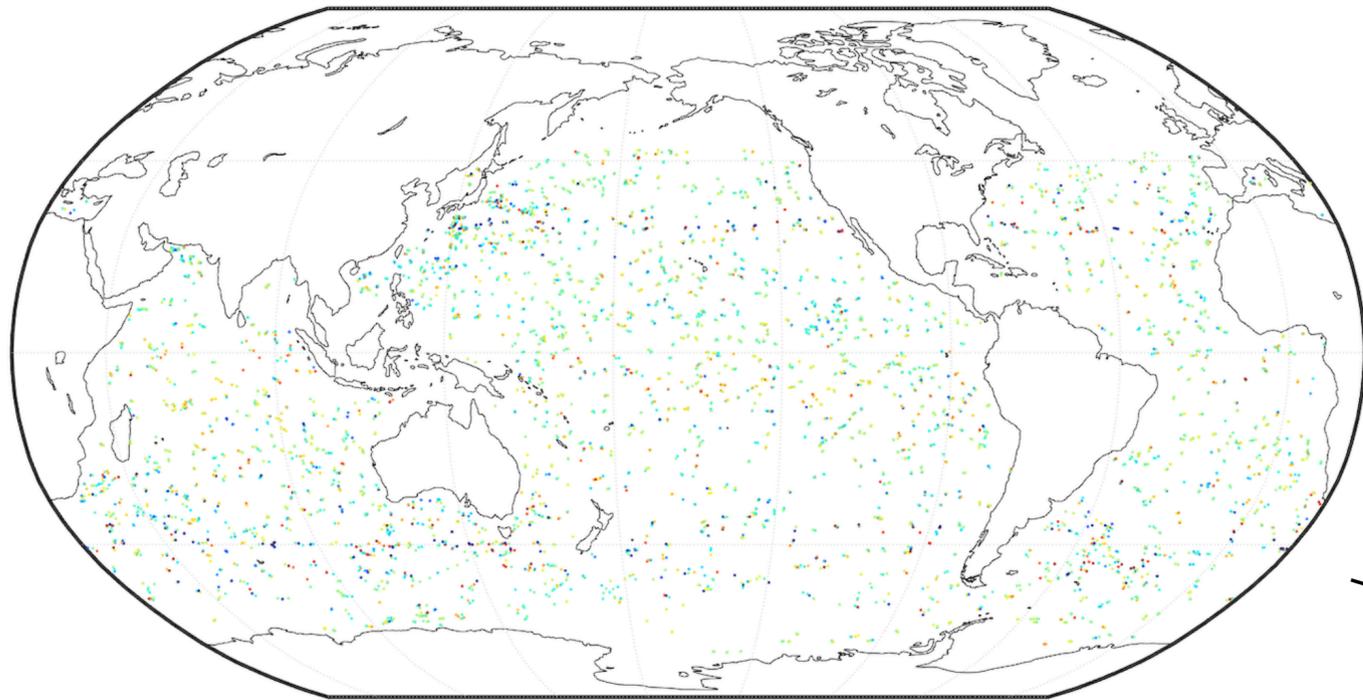
$$\text{Var}(\text{OHC}_{\text{bot}}|\text{data})$$



$$\text{Var}(\text{OHC}_{\text{total}}|\text{data})$$

We can improve the uncertainty estimates by also modeling the **correlation**.

A bivariate GP model accounts for cross-layer correlation



Temperature residuals

Latitude
Longitude

Date

Nugget effect

$$\begin{bmatrix} y_{\text{top}} \\ y_{\text{bot}} \end{bmatrix}_{i,j} = f_i \left(\begin{bmatrix} x_{\text{top}} \\ x_{\text{bot}} \end{bmatrix}_{i,j}, \begin{bmatrix} t_{\text{top}} \\ t_{\text{bot}} \end{bmatrix}_{i,j} \right) + \begin{bmatrix} \epsilon_{\text{top}} \\ \epsilon_{\text{bot}} \end{bmatrix}_{i,j}$$

$$f_i \stackrel{\text{iid}}{\sim} \text{GP} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{K}(x_1, t_1, x_2, t_2; \boldsymbol{\theta}) \right)$$

(Covariance function)

$$\begin{bmatrix} \epsilon_{\text{top}} \\ \epsilon_{\text{bot}} \end{bmatrix} \stackrel{\text{iid}}{\sim} \text{N} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma}_{\epsilon}(\boldsymbol{\theta}_{\epsilon}) \right)$$

A bivariate GP model accounts for cross-layer correlation

Marginal covariance
(Kuusela and Stein 2018)

$$\mathbf{K}_{ii}(z_1, z_2; \theta) = \frac{\delta_i^2}{\sqrt{|\Theta_i|}} \exp(-\sqrt{(z_1 - z_2)^T \Theta_i^{-1} (z_1 - z_2)})$$

GP variance

Space-time distance
w/ length scale parameters

Cross-covariance
(Kleiber and Nychka 2012)

$$\mathbf{K}_{\text{top,bot}}(z_1, z_2; \theta) = \beta \frac{\delta_{\text{top}} \delta_{\text{bot}}}{\sqrt{|\Theta_{\text{top,bot}}|}} \exp(-\sqrt{(z_1 - z_2)^T \Theta_{\text{top,bot}}^{-1} (z_1 - z_2)})$$

(Cross) correlation