Toward improved OHC mapping and uncertainty quantification by modeling vertical spatio-temporal dependence

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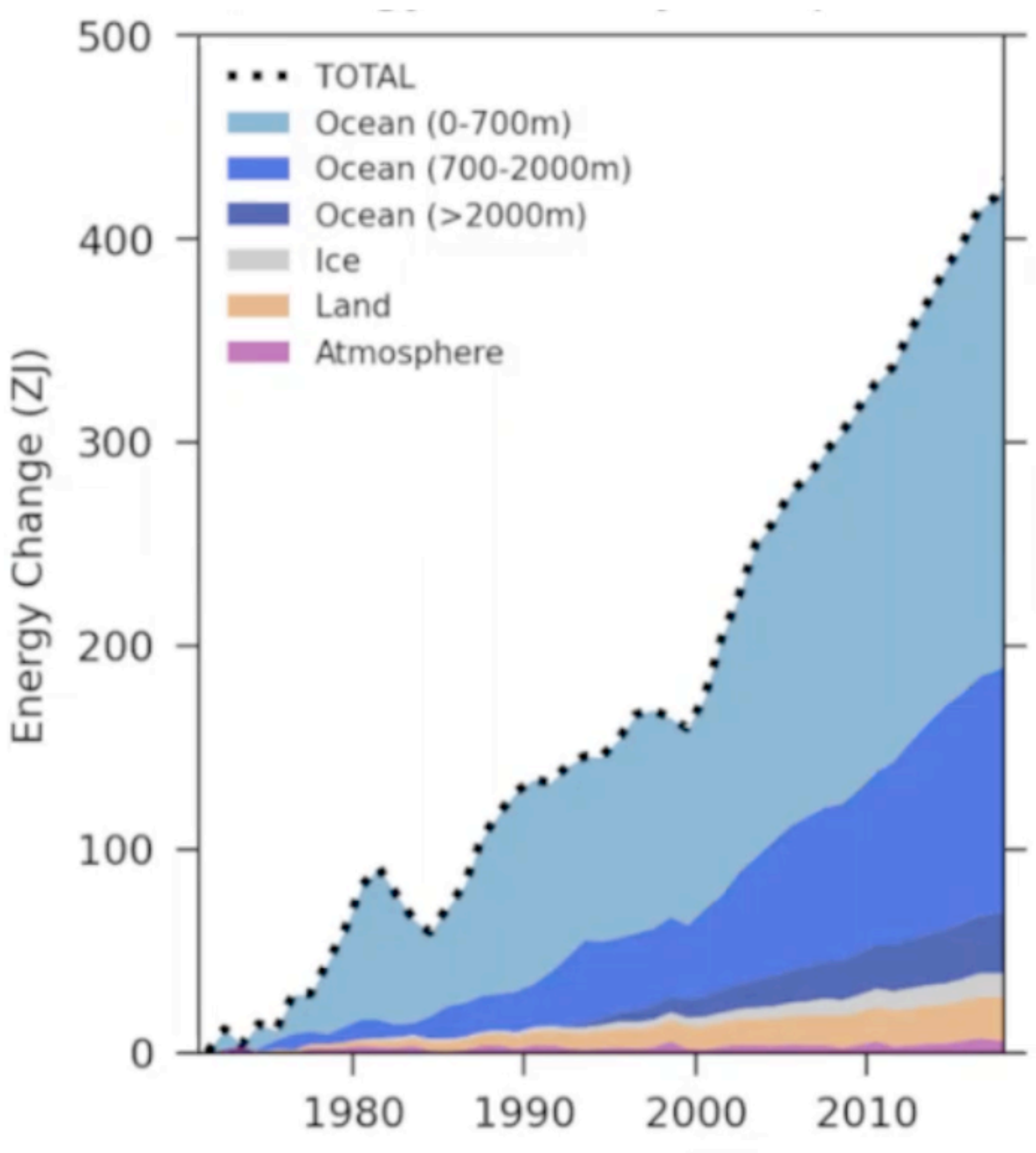
28 June 2024

Carnegie Mellon University
Statistics & Data Science

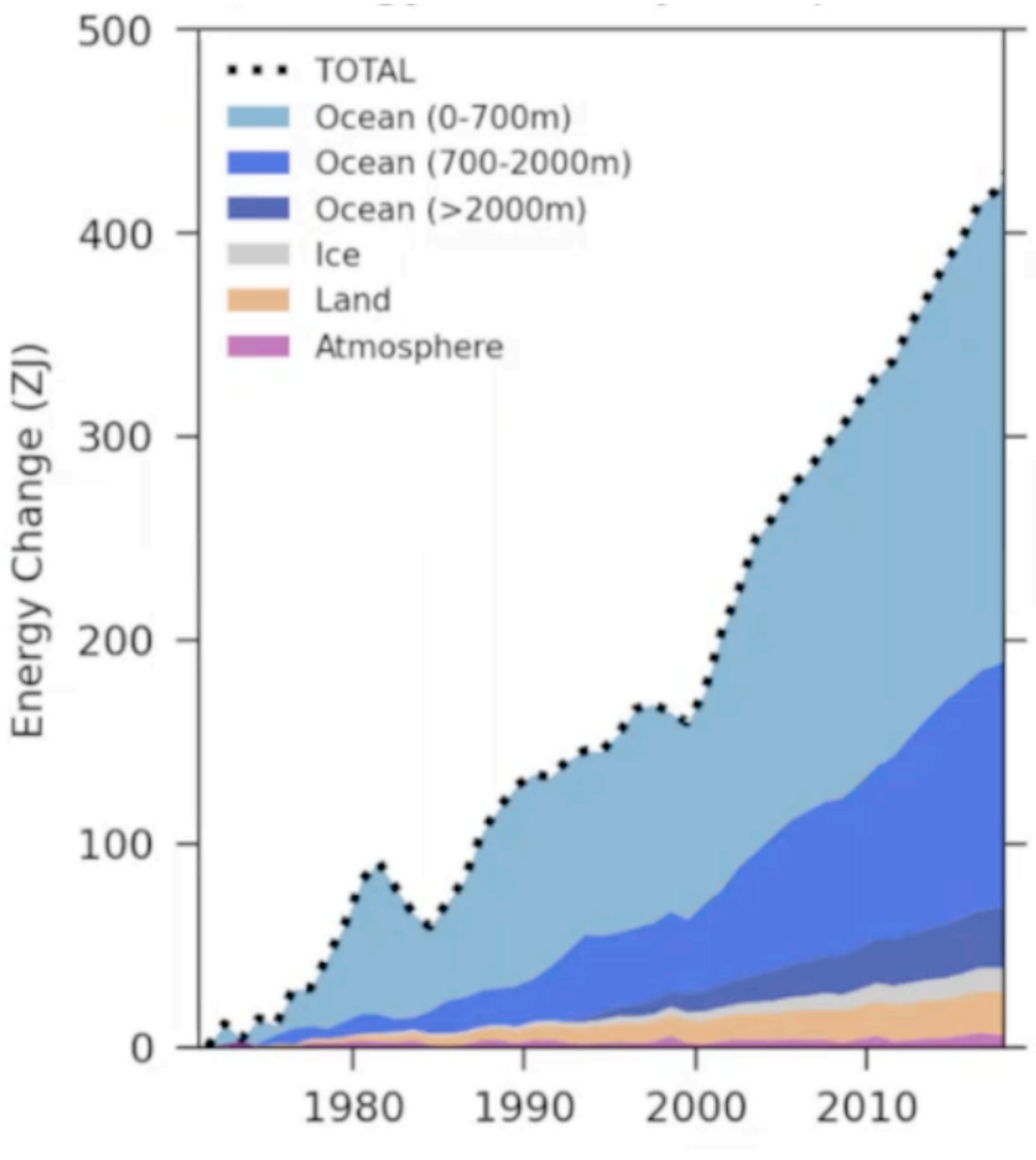
University of Colorado
Boulder

Most of the excess heat in the climate system has been stored in the ocean



(IPCC AR6)

Changes in OHC contribute to extreme climate events

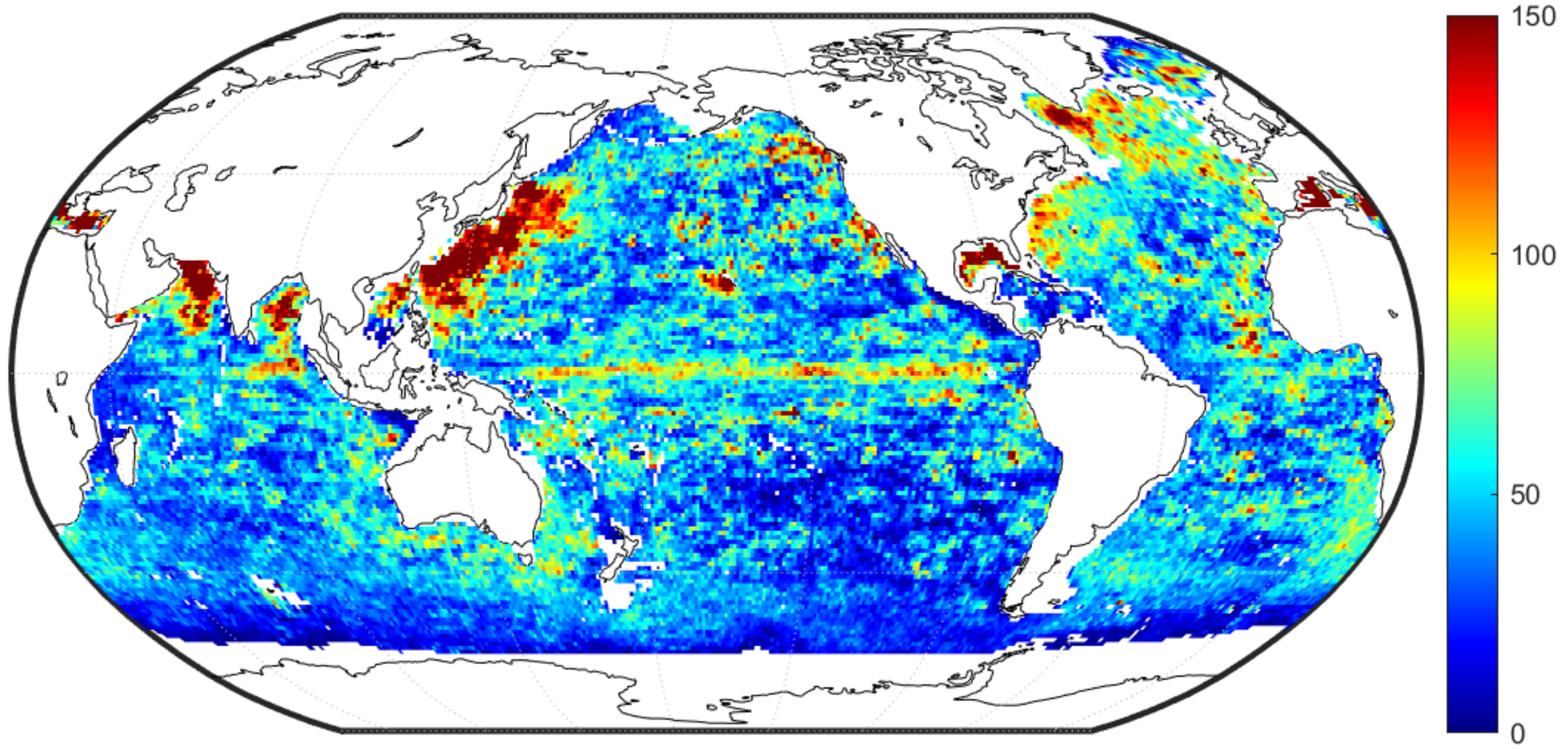


(IPCC AR6)

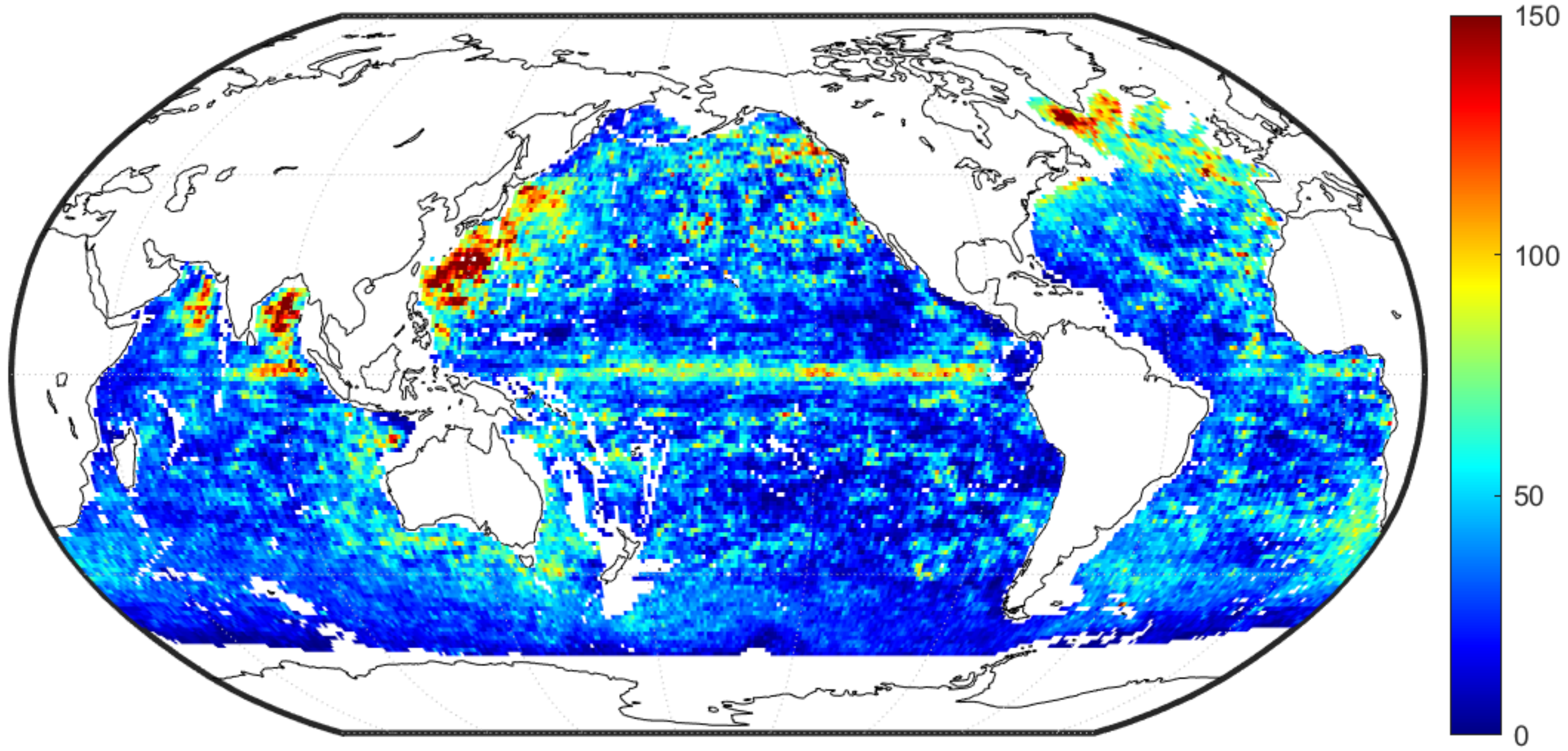
Intensified tropical storms

Rising sea levels

We use Argo data (2004-2022) to estimate 15-1850 dbar OHC

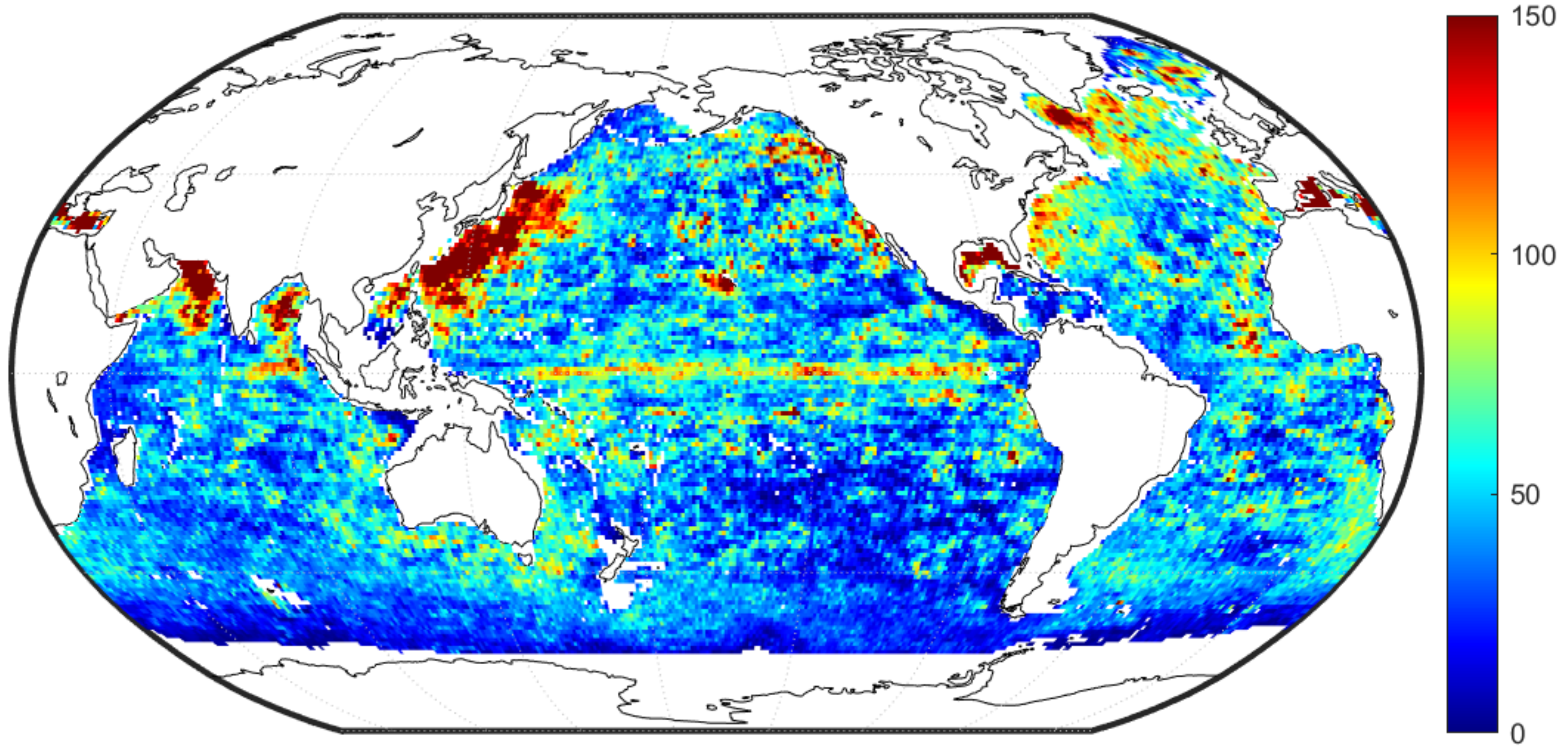


Top layer profiles (15-975 dbar)

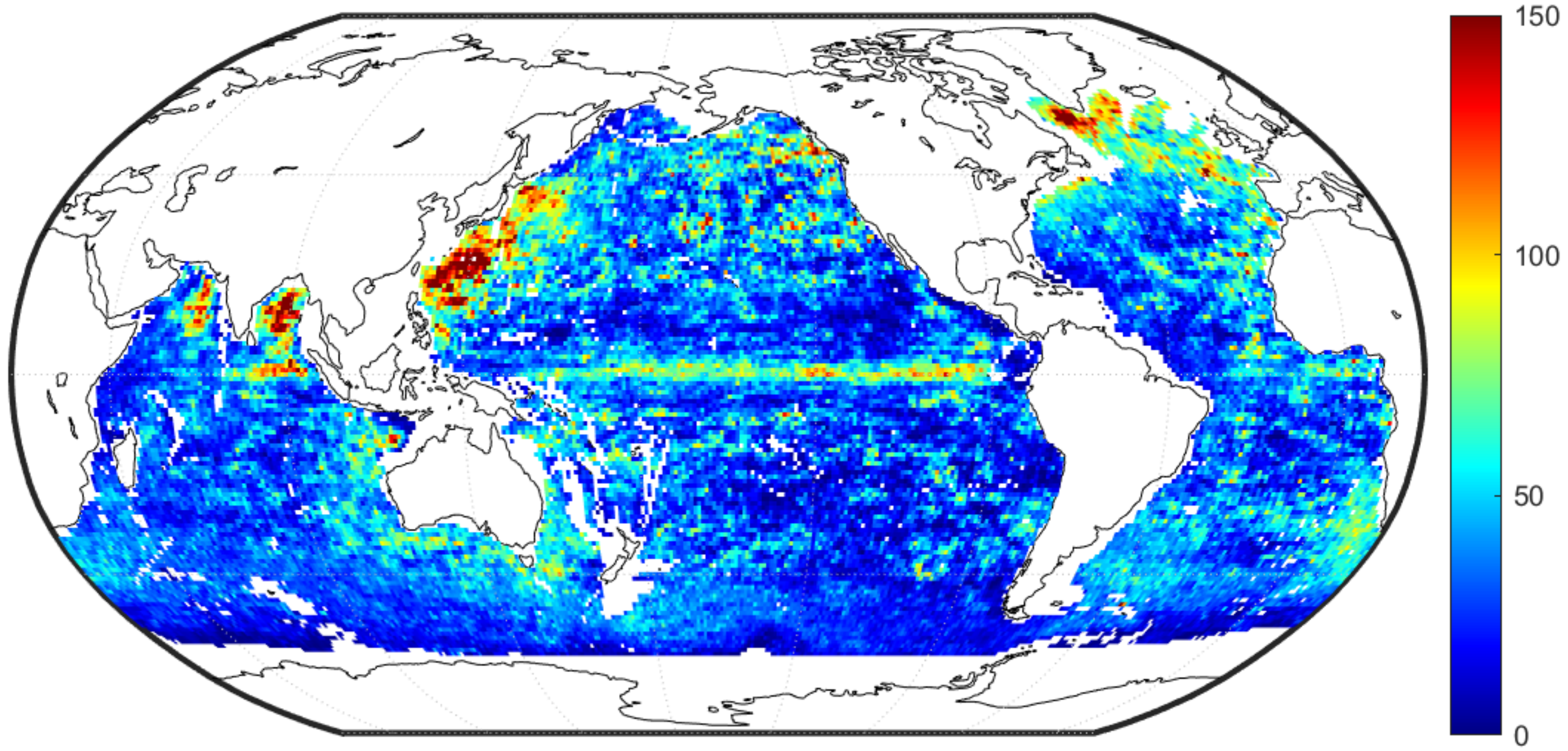


Bottom layer profiles (975-1850 dbar)

We use Argo data (2004-2022) to estimate 15-1850 dbar OHC



Top layer profiles (15-975 dbar)



Bottom layer profiles (975-1850 dbar)

Due to having fewer observations deeper in the water column, we model the OHC in the top and bottom layers **separately.**

How do we model OHC from sparse observations?

1. Integrate the **vertical dimension** (consider two layers and integrate Argo profiles in each layer)

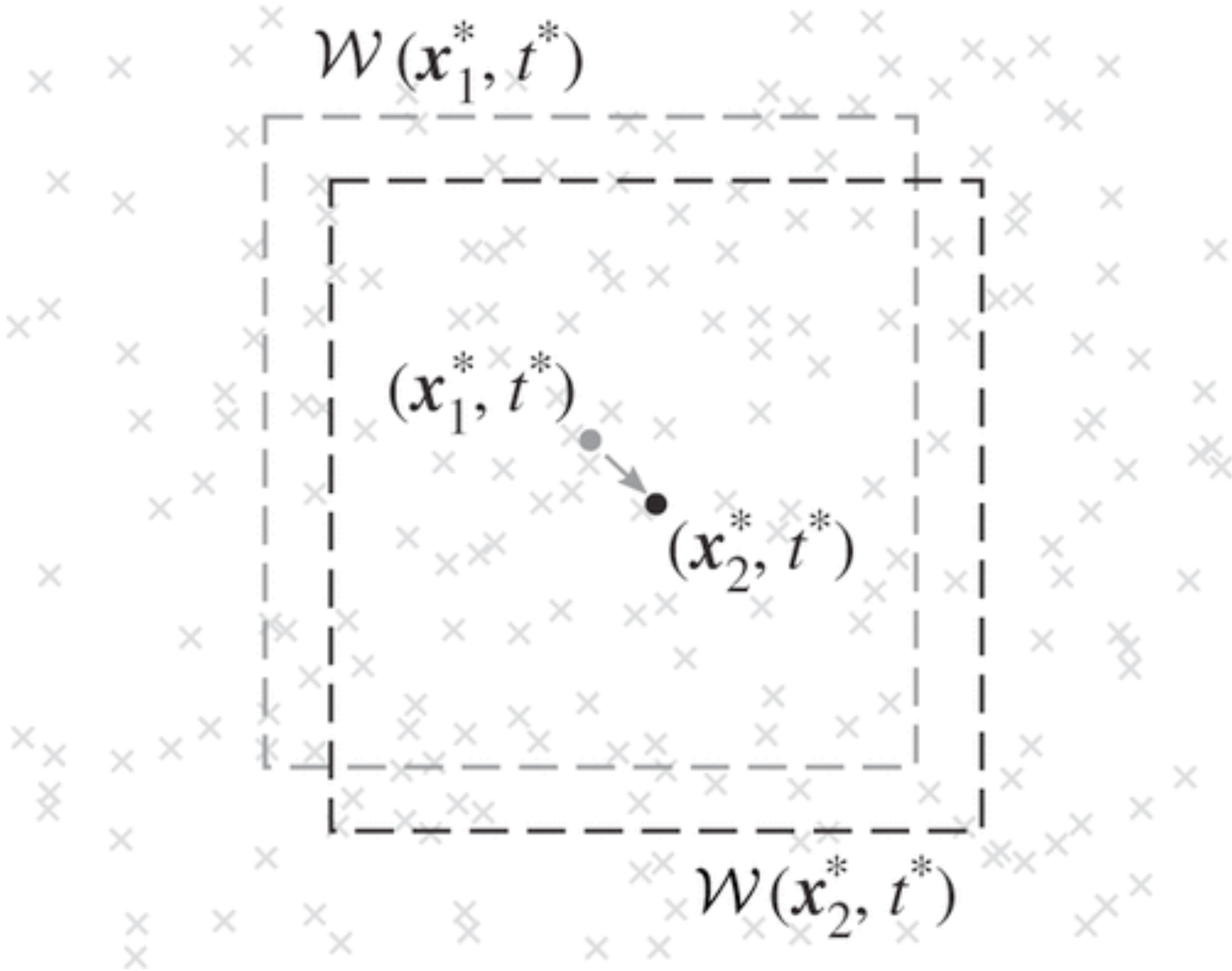
2. Model **horizontal and temporal dimension**

3. Estimate **uncertainties** based on (1) and (2)

We can estimate a gridded temperature field with GP regression

1. Model temperature mean field: local least squares regression (Roemmich and Gilson 2009) + linear time trend

2. Model residuals: **locally stationary** Gaussian process (GP) regression (Kuusela and Stein 2018)



How do we estimate uncertainties for the total (top + bottom layers) OHC?

$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{total}}|\mathrm{data})$$

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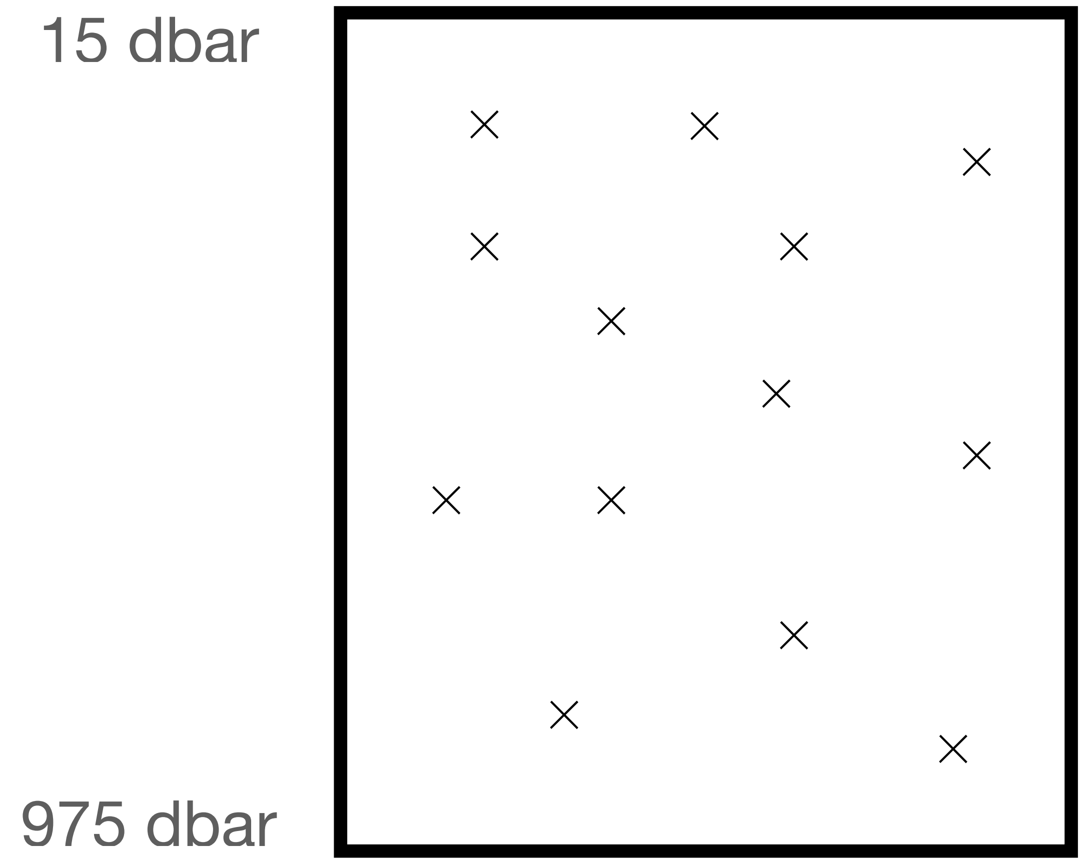
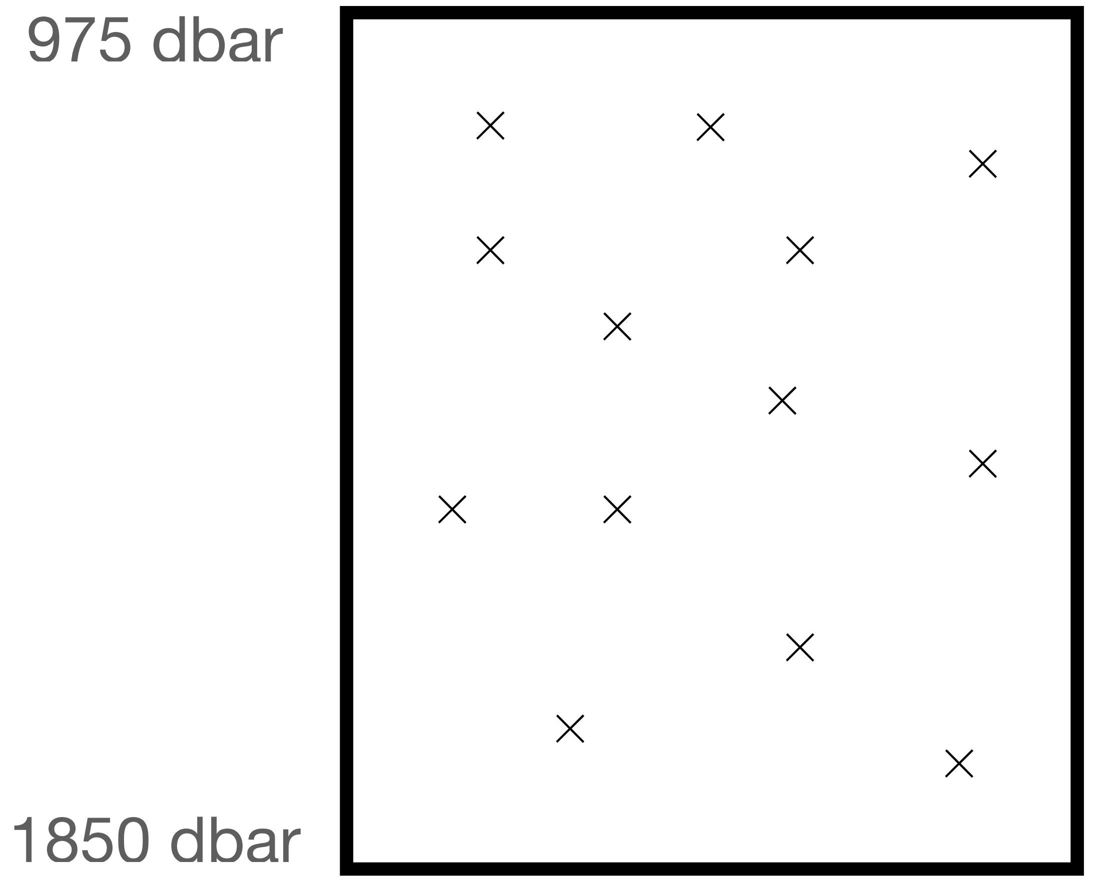


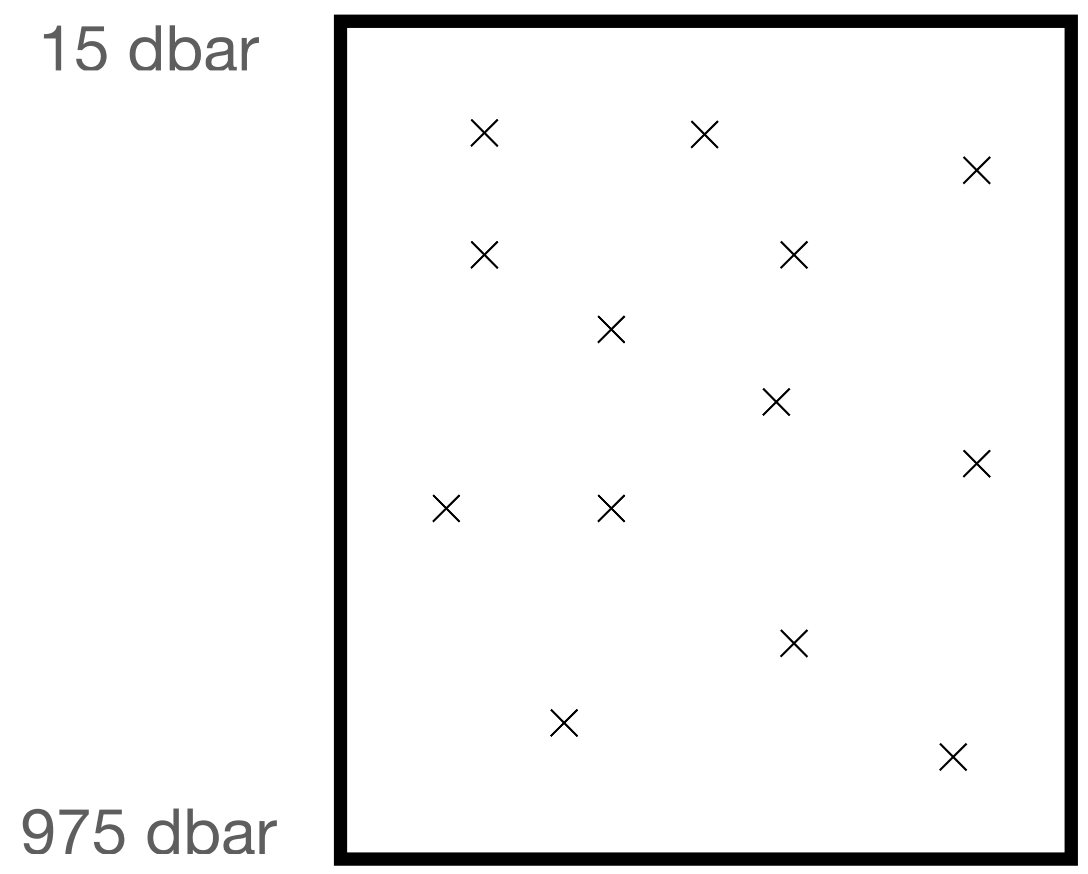
$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{top}}|\mathrm{data})$$

$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{total}}|\mathrm{data})$$



$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})$$

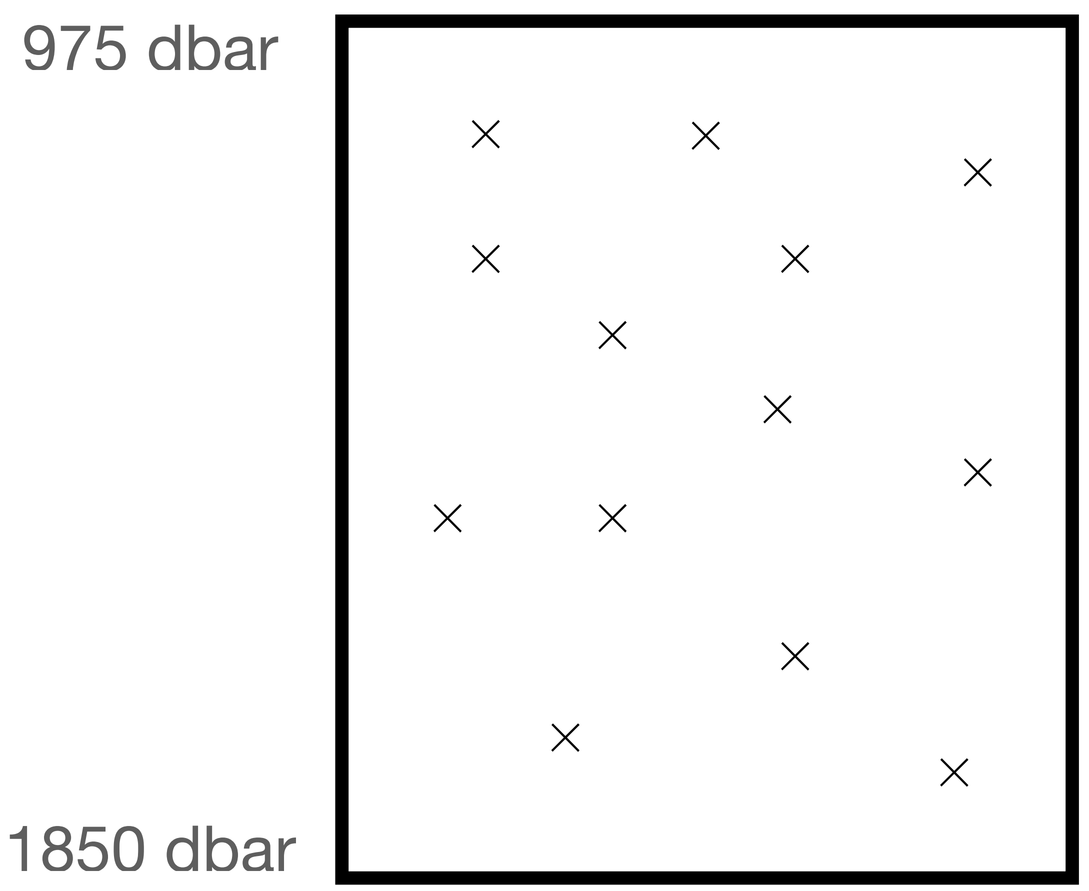
How do we estimate uncertainties for the total (top + bottom layers) OHC?



$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{top}}|\mathrm{data})$$

$$\cancel{\mathrm{Var}(\mathrm{OHC}_{\mathrm{total}}|\mathrm{data})}$$

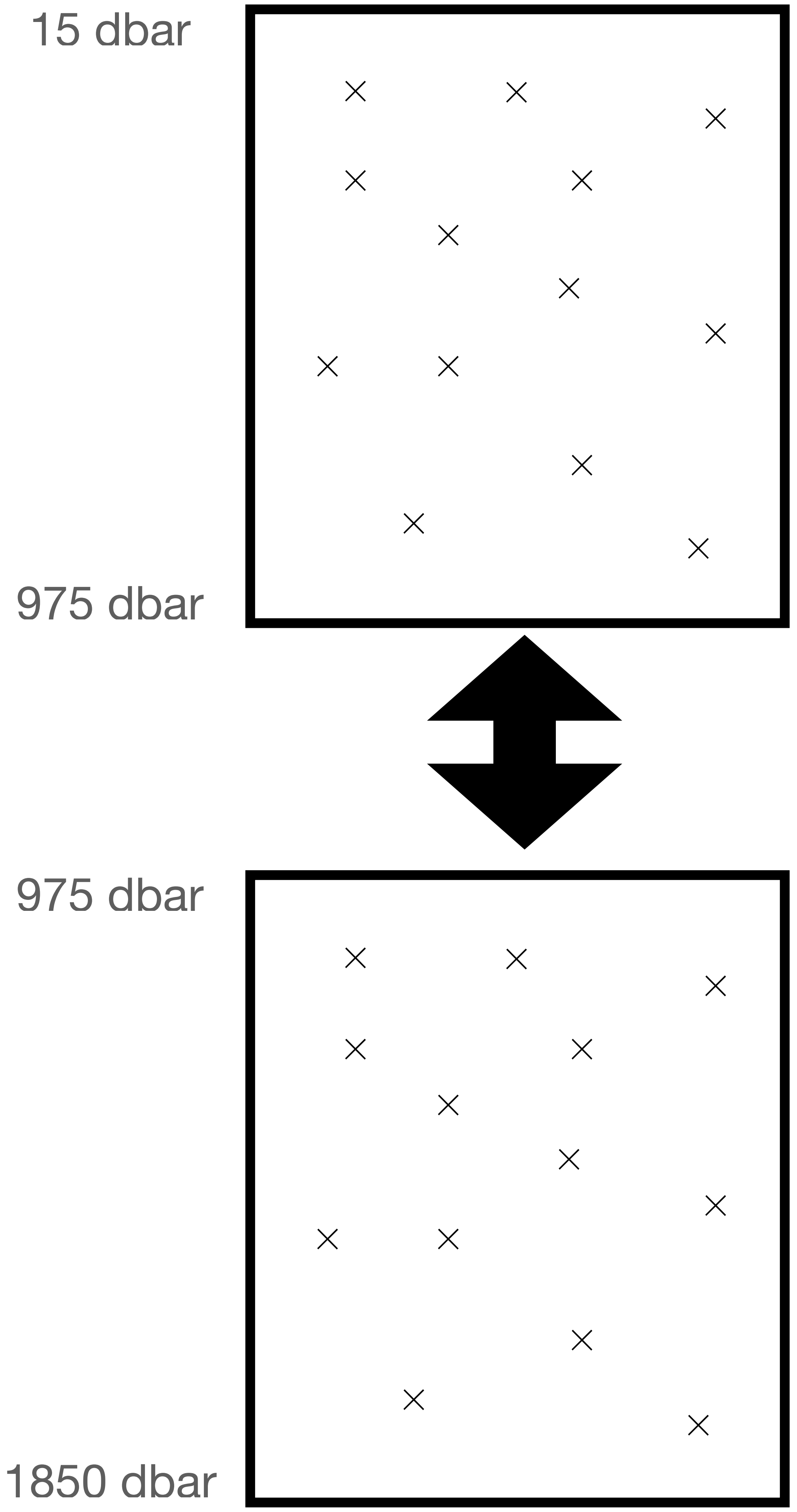
$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{top}}|\mathrm{data}) + \mathrm{Var}(\mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})$$



$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})$$

Summing the variances for each layer **underestimates** the uncertainties of the total OHC.

How do we estimate uncertainties for the total (top + bottom layers) OHC?

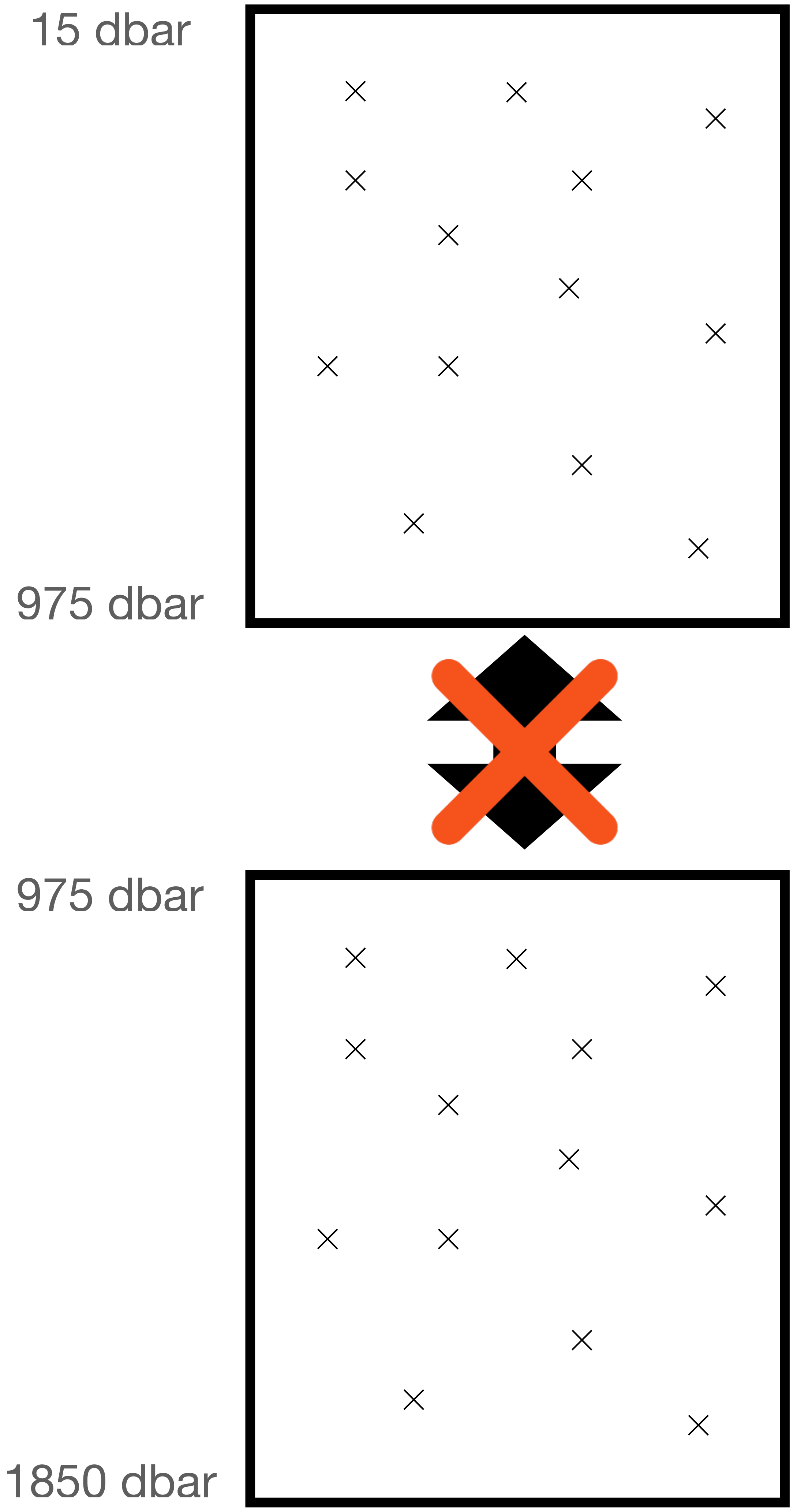


$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{top}}|\mathrm{data})$$

$$2\mathrm{Cov}(\mathrm{OHC}_{\mathrm{top}}, \mathrm{OHC}_{\mathrm{bot}}|\mathrm{data}) \longrightarrow \mathrm{Var}(\mathrm{OHC}_{\mathrm{total}}|\mathrm{data})$$

$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})$$

How do we estimate uncertainties for the total (top + bottom layers) OHC?



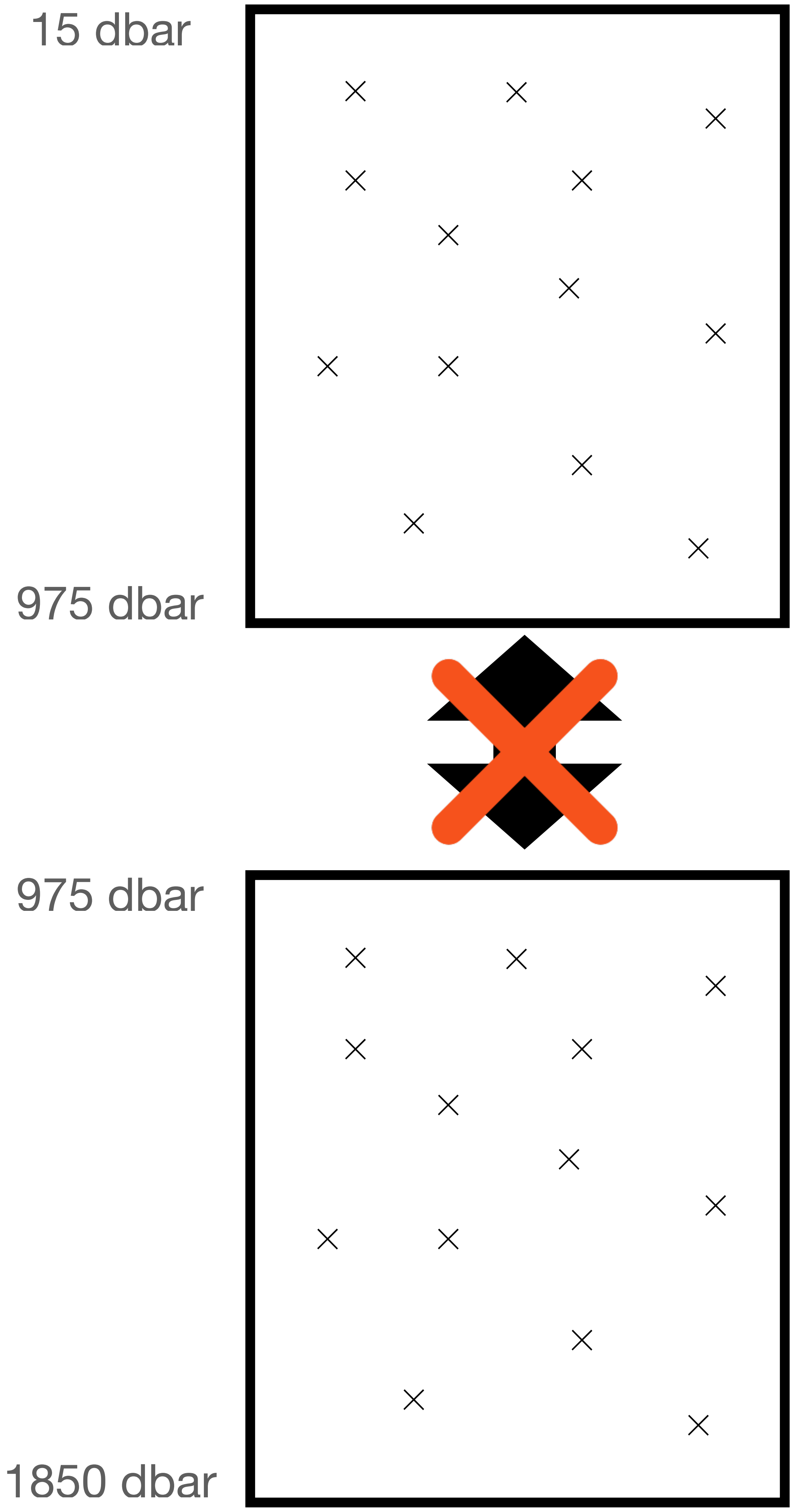
$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{top}}|\mathrm{data})$$

$$2\mathrm{Cov}(\mathrm{OHC}_{\mathrm{top}}, \mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})$$

$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})$$

$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{total}}|\mathrm{data})$$

How do we estimate uncertainties for the total (top + bottom layers) OHC?



$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{top}}|\mathrm{data})$$

$$2\mathrm{Cov}(\mathrm{OHC}_{\mathrm{top}}, \mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})$$

$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})$$

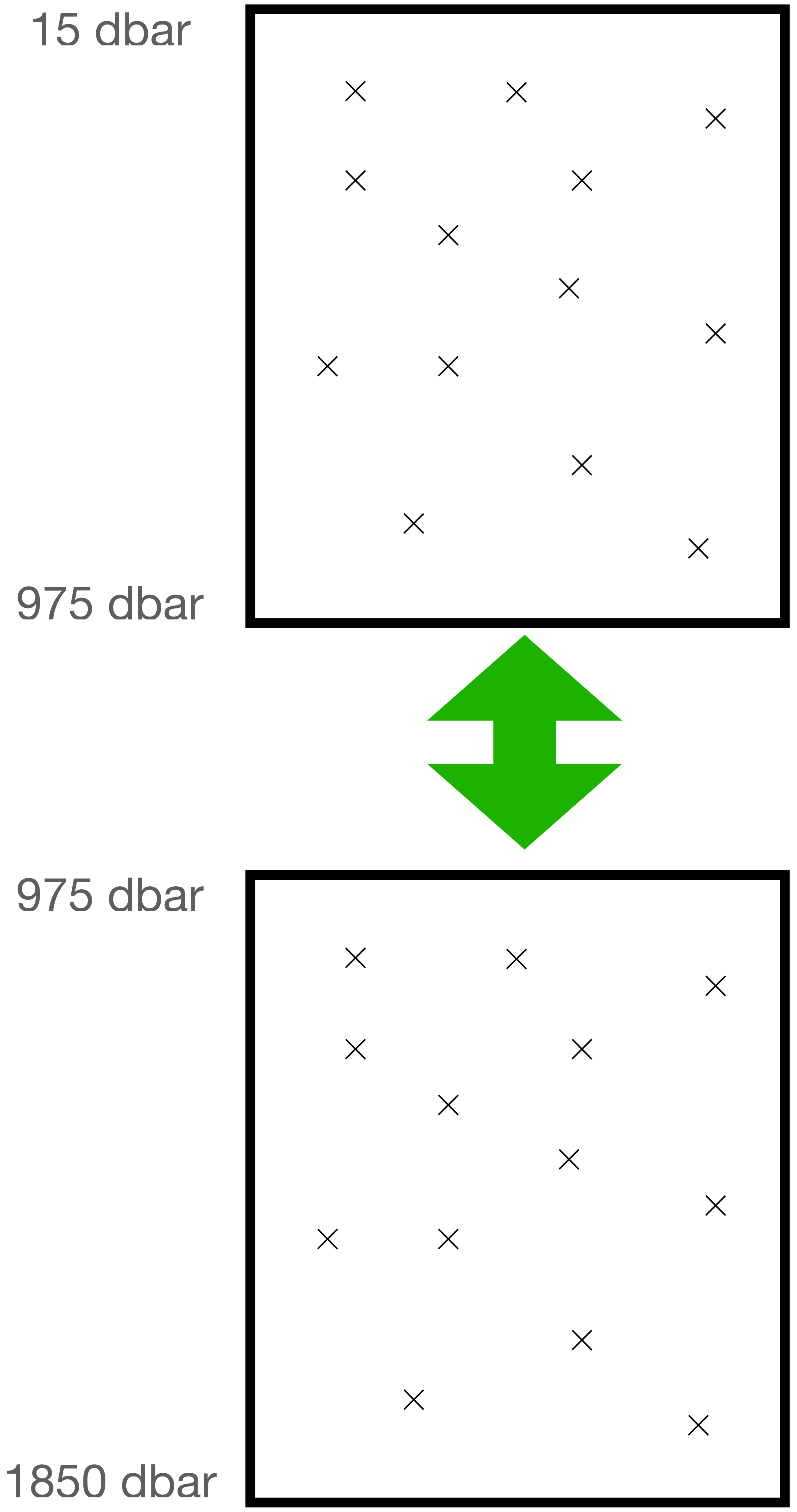
$$\cancel{\mathrm{Var}(\mathrm{OHC}_{\mathrm{total}}|\mathrm{data})}$$

(conservative upper bound)

$$(\sqrt{\mathrm{Var}(\mathrm{OHC}_{\mathrm{top}}|\mathrm{data})} + \sqrt{\mathrm{Var}(\mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})})^2$$

Squaring the sum of the standard deviations for each layer **overestimates** the uncertainties of the total OHC.

How do we estimate uncertainties for the total (top + bottom layers) OHC?



$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{top}}|\mathrm{data})$$

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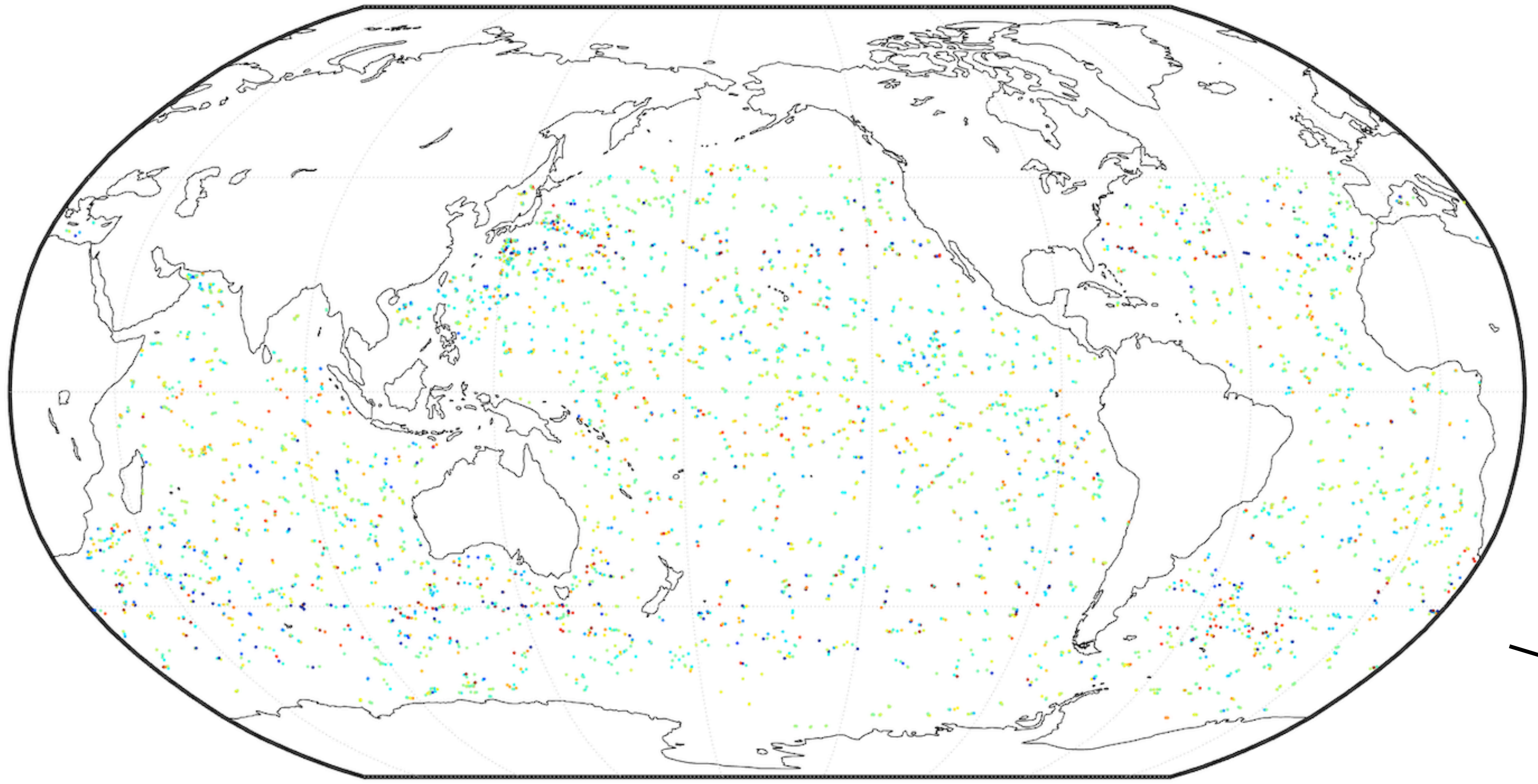
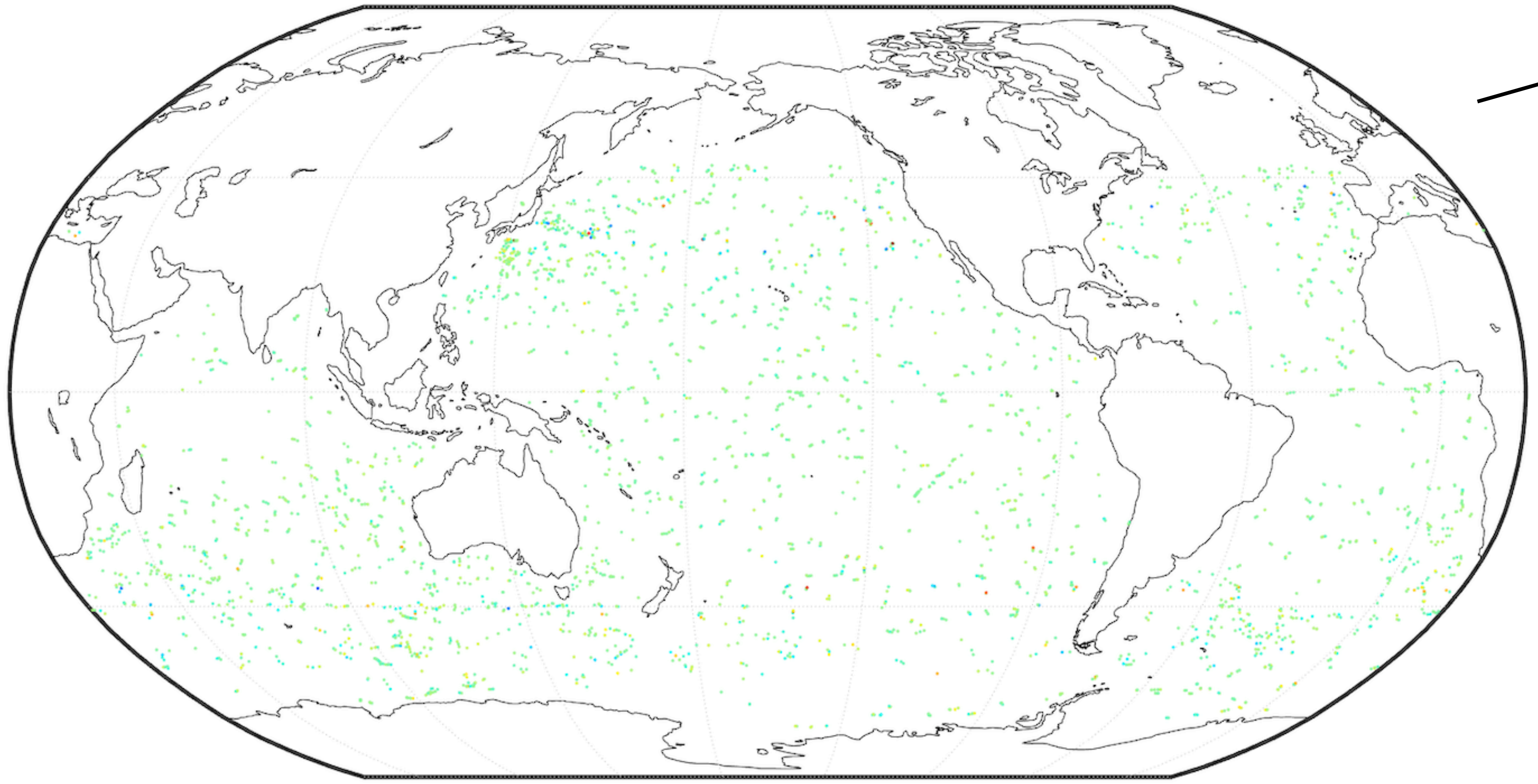
$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})$$

$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{total}}|\mathrm{data})$$

We can improve the uncertainty estimates by also modeling the **correlation.**

A bivariate GP model accounts for cross-layer correlation

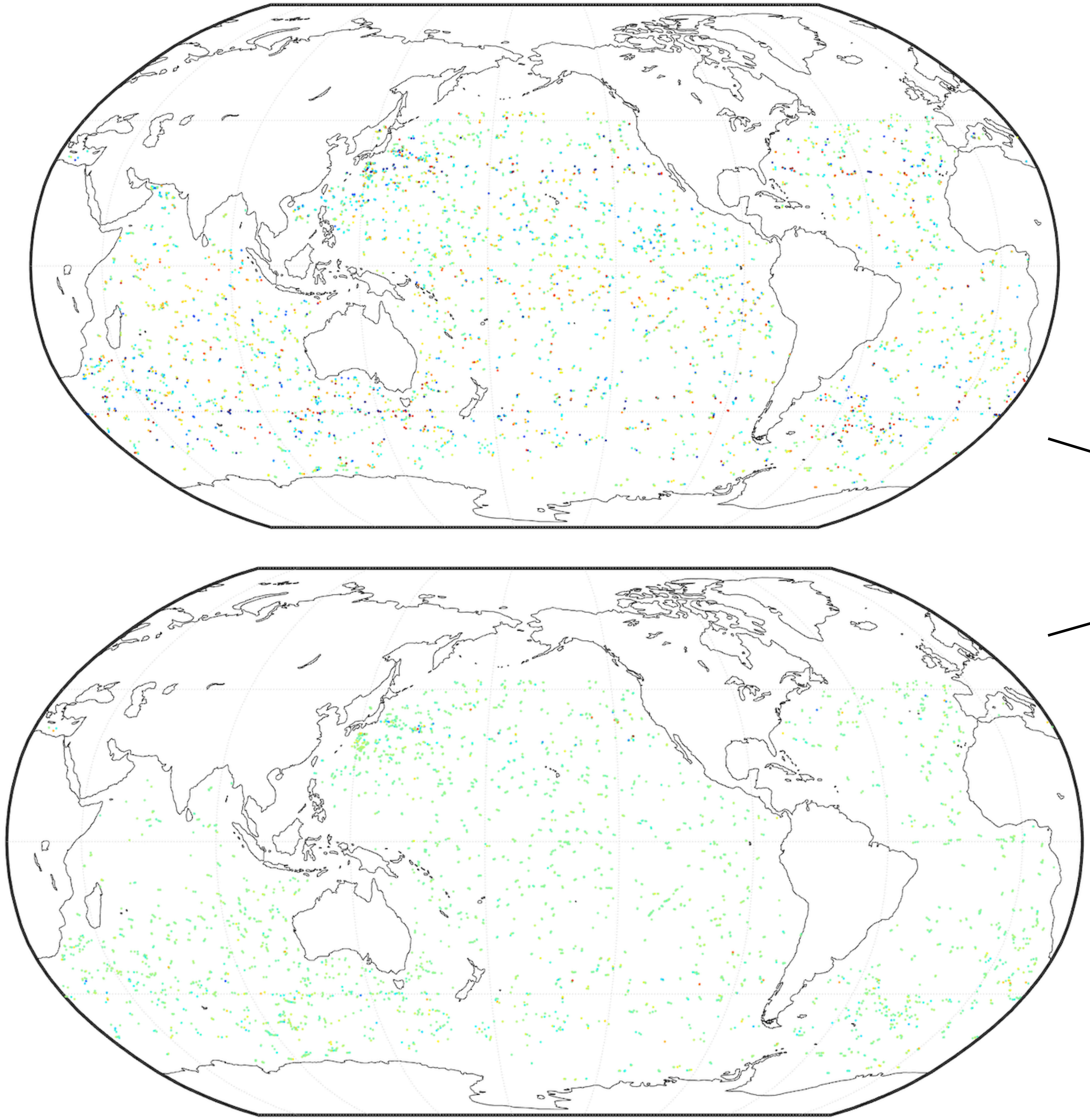




Temperature residuals

$$\begin{bmatrix} y_{\text{top}} \\ y_{\text{bot}} \end{bmatrix}_{i,j}$$

A bivariate GP model accounts for cross-layer correlation

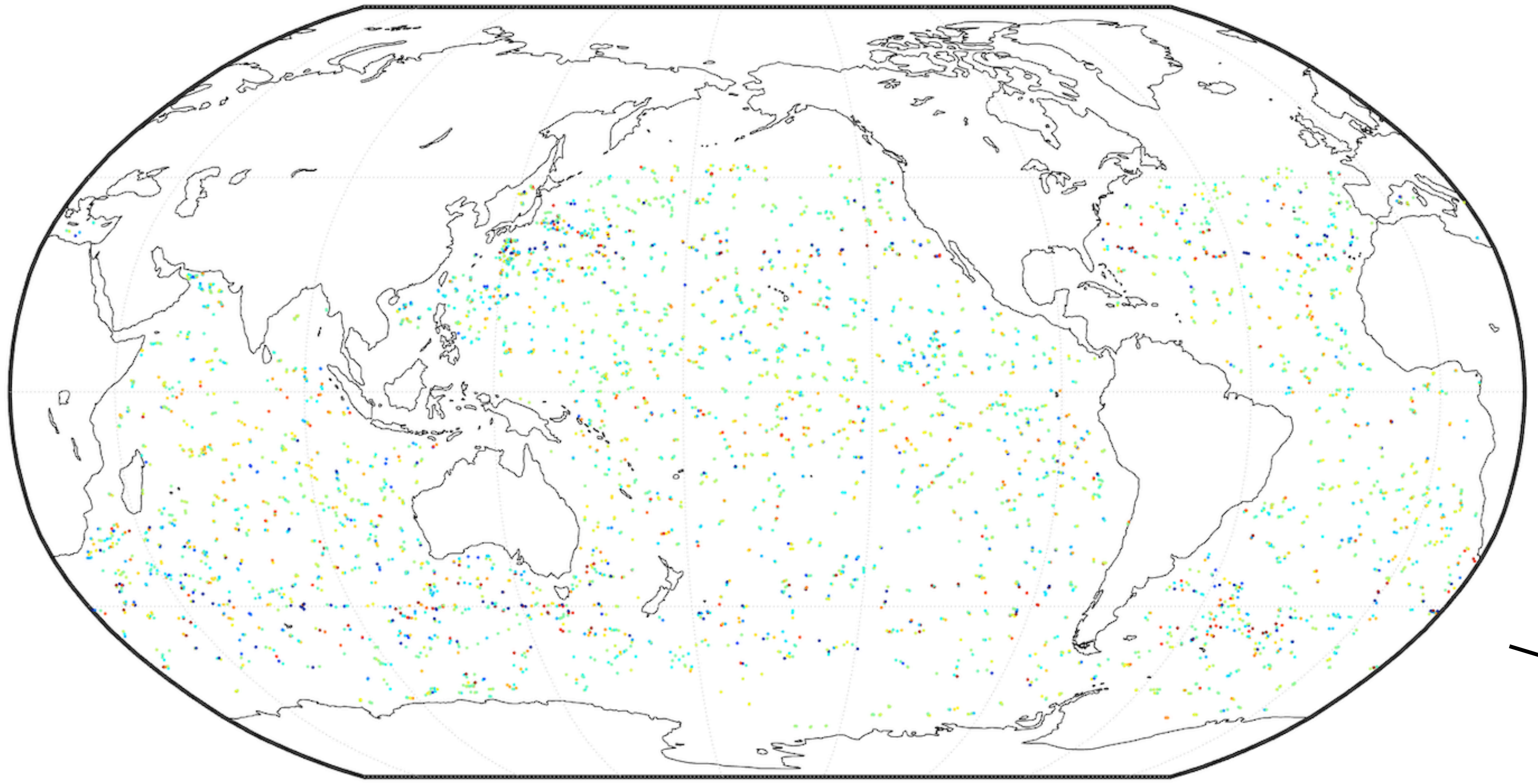


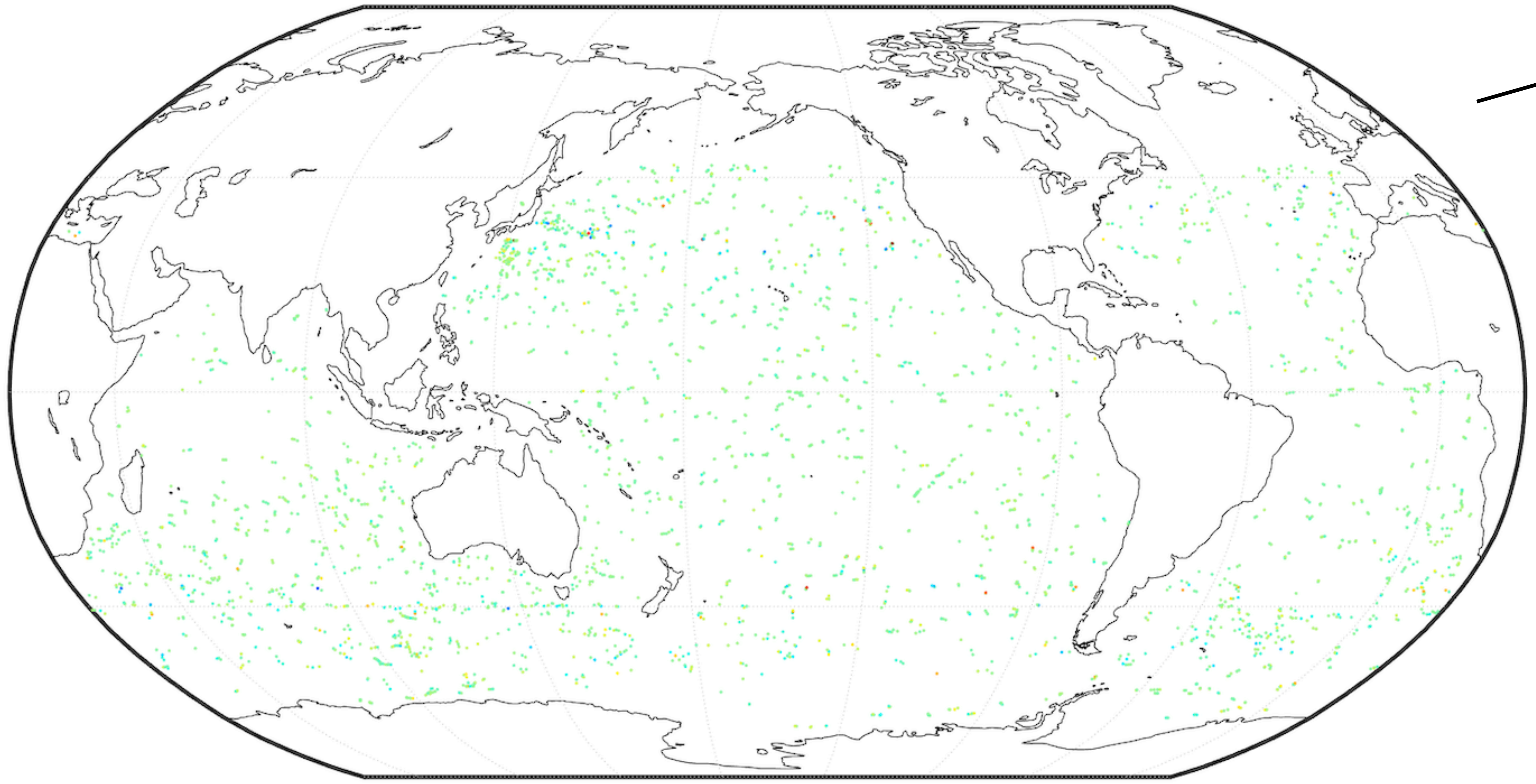
Temperature residuals — Latitude Longitude — Date

$$\begin{bmatrix} y_{\text{top}} \\ y_{\text{bot}} \end{bmatrix}_{i,j} = f_i\left(\begin{bmatrix} x_{\text{top}} \\ x_{\text{bot}} \end{bmatrix}_{i,j}, \begin{bmatrix} t_{\text{top}} \\ t_{\text{bot}} \end{bmatrix}_{i,j}\right)$$

$$f_i \overset{\text{iid}}{\sim} \text{GP}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{K}(x_1, t_1, x_2, t_2; \boldsymbol{\theta})\right)$$

A bivariate GP model accounts for cross-layer correlation





Temperature residuals | **Latitude Longitude** | **Date** | **Nugget effect**

$$\begin{bmatrix} y_{\text{top}} \\ y_{\text{bot}} \end{bmatrix}_{i,j} = f_i\left(\begin{bmatrix} x_{\text{top}} \\ x_{\text{bot}} \end{bmatrix}_{i,j}, \begin{bmatrix} t_{\text{top}} \\ t_{\text{bot}} \end{bmatrix}_{i,j}\right) + \begin{bmatrix} \epsilon_{\text{top}} \\ \epsilon_{\text{bot}} \end{bmatrix}_{i,j}$$

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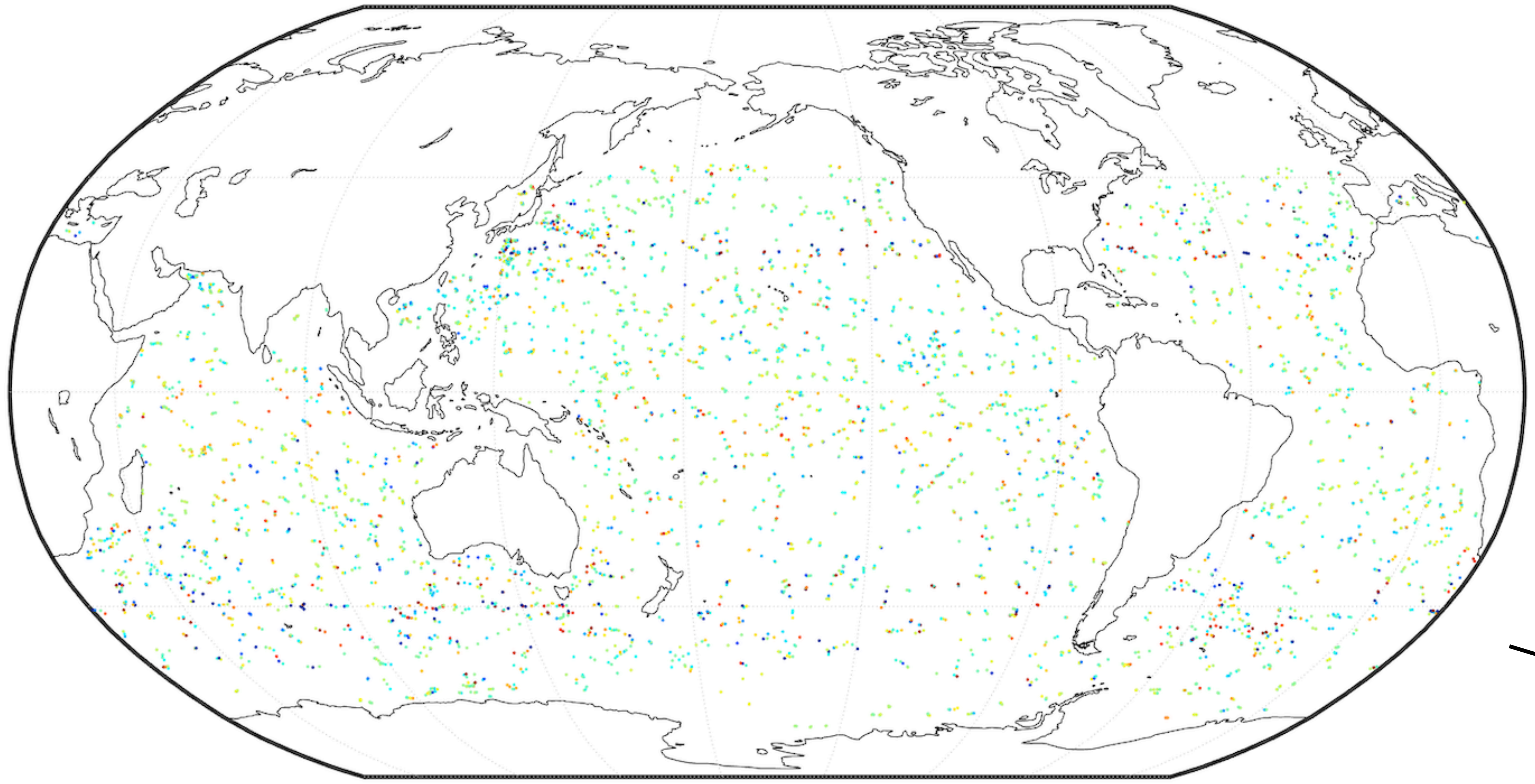
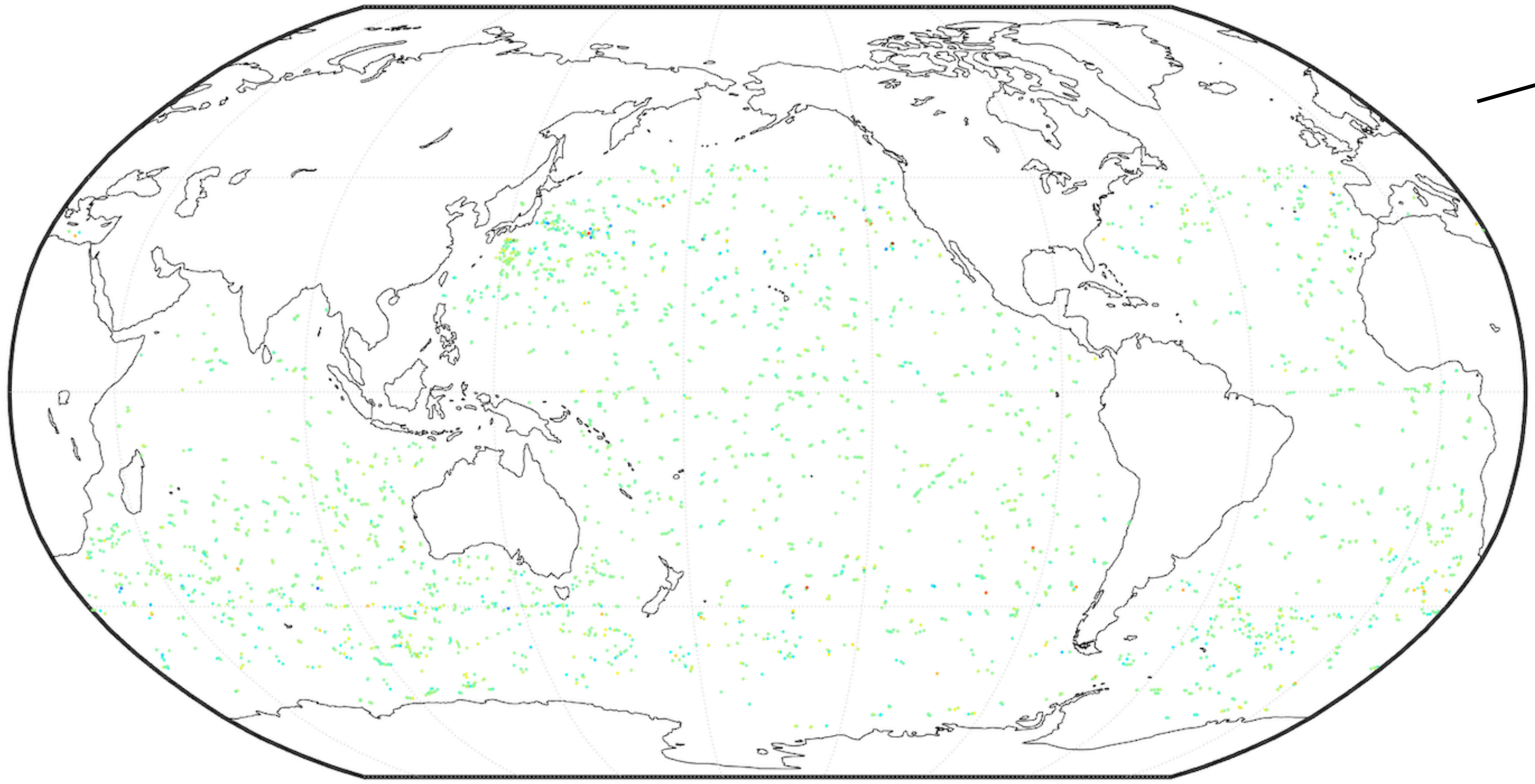
$$\begin{bmatrix} \epsilon_{\text{top}} \\ \epsilon_{\text{bot}} \end{bmatrix} \overset{\text{iid}}{\sim} \text{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma}_{\epsilon}(\boldsymbol{\theta}_{\epsilon})\right)$$

A bivariate GP model accounts for cross-layer correlation





Temperature residuals		Latitude Longitude	Date		Nugget effect

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$$f_i \overset{\text{iid}}{\sim} \mathrm{GP}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{K}(x_1, t_1, x_2, t_2; \boldsymbol{\theta})\right)$$

(Covariance function)

$$\begin{bmatrix} \epsilon_{\text{top}} \\ \epsilon_{\text{bot}} \end{bmatrix} \overset{\text{iid}}{\sim} \mathrm{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{\Sigma}_{\epsilon}(\boldsymbol{\theta}_{\epsilon})\right)$$

A bivariate GP model accounts for cross-layer correlation

Marginal covariance
(Kuusela and Stein 2018)

$$\boldsymbol{K}_{\text{ii}}(\boldsymbol{z_1}, \boldsymbol{z_2}; \boldsymbol{\theta})$$

A bivariate GP model accounts for cross-layer correlation

Marginal covariance
(Kuusela and Stein 2018)

$$\boldsymbol{K}_{\mathrm{ii}}(\boldsymbol{z_1}, \boldsymbol{z_2}; \boldsymbol{\theta}) = \frac{\delta_i^2}{\sqrt{|\boldsymbol{\Theta}_i|}}$$

GP variance

A bivariate GP model accounts for cross-layer correlation

Marginal covariance
(Kuusela and Stein 2018)

$$\boldsymbol{K}_{\mathrm{ii}}(\boldsymbol{z_1}, \boldsymbol{z_2}; \boldsymbol{\theta}) = \frac{\delta_i^2}{\sqrt{|\boldsymbol{\Theta}_i|}} \exp(-\sqrt{(\boldsymbol{z_1} - \boldsymbol{z_2})^{\mathrm{T}} \boldsymbol{\Theta}_i^{-1} (\boldsymbol{z_1} - \boldsymbol{z_2})})$$

GP variance

Space-time distance
w/ length scale parameters

A bivariate GP model accounts for cross-layer correlation

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(Kuusela and Stein 2018)

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GP variance

Space-time distance
w/ length scale parameters

Cross-covariance
(Kleiber and Nychka 2012)

$$\boldsymbol{K}_{\mathrm{top,bot}}(\boldsymbol{z_1}, \boldsymbol{z_2}; \boldsymbol{\theta}) = \beta \frac{\delta_{\mathrm{top}} \delta_{\mathrm{bot}}}{\sqrt{|\boldsymbol{\Theta}_{\mathrm{top,bot}}|}} \exp\left(-\sqrt{(\boldsymbol{z_1} - \boldsymbol{z_2})^{\mathrm{T}} \boldsymbol{\Theta}_{\mathrm{top,bot}}^{-1} (\boldsymbol{z_1} - \boldsymbol{z_2})}\right)$$

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GP variance

Space-time distance w/ length scale parameters

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(Cross) correlation

Obtaining uncertainties is facilitated by local conditional simulations

full field | data ?

Obtaining uncertainties is facilitated by local conditional simulations

full field | data - **multivariate normal** with conditional covariance Σ_i

(parameterized by estimated GP variance, length scales, cross-correlation)

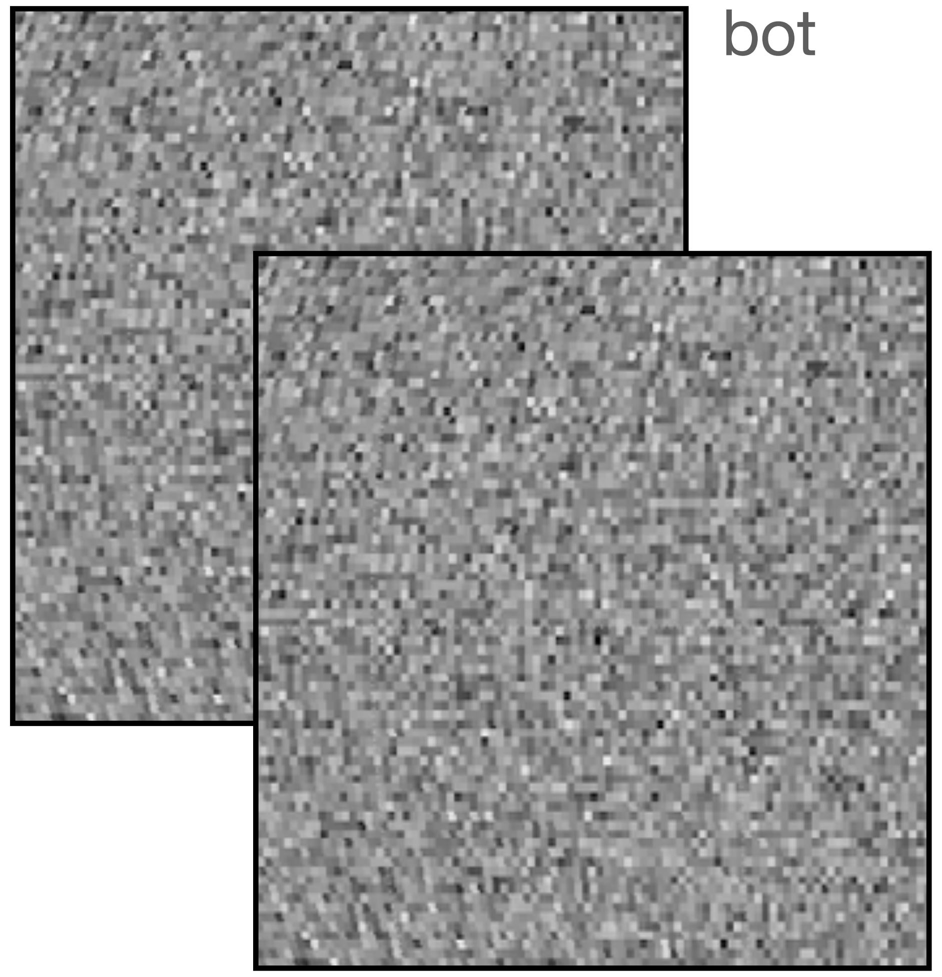
Obtaining uncertainties is facilitated by local conditional simulations

full field | data - **multivariate normal** with conditional covariance Σ_i

(parameterized by estimated GP variance, length scales, cross-correlation)

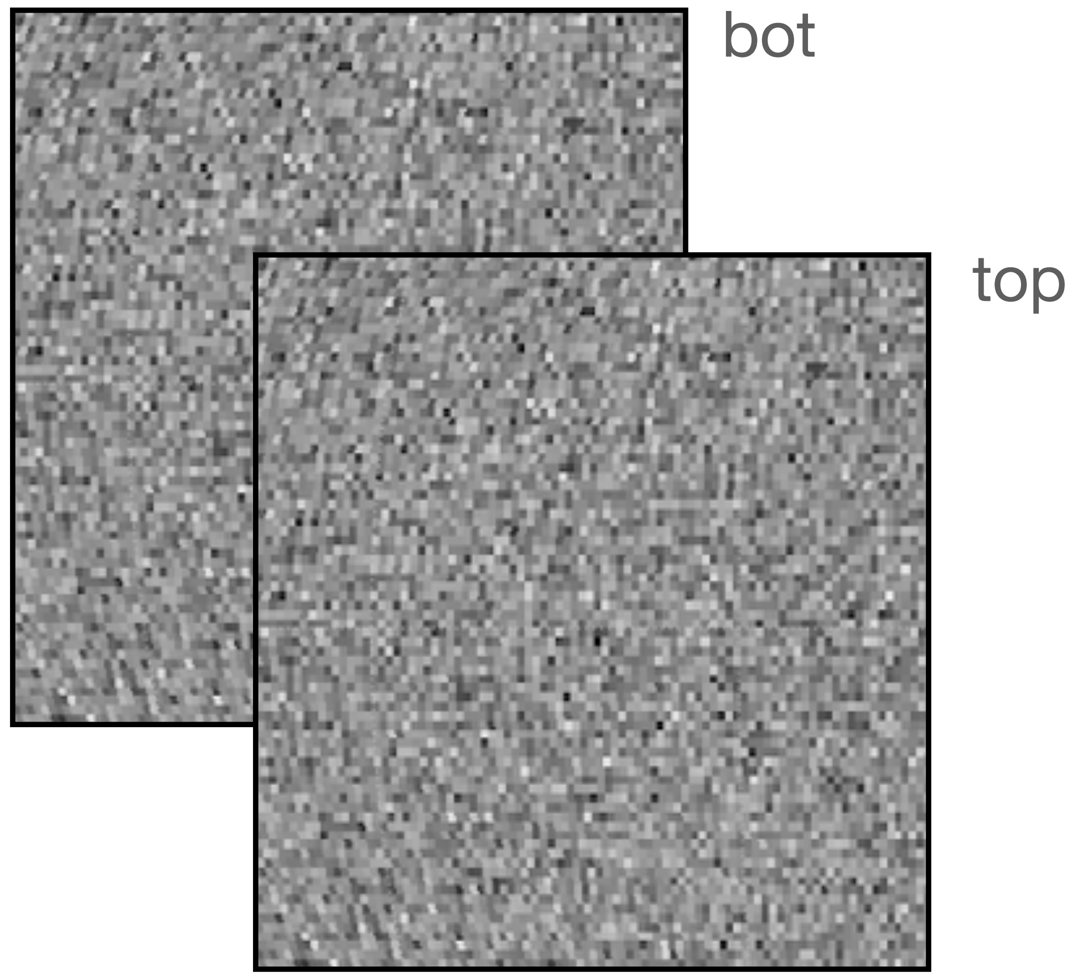
Local conditional simulations! (extension of Nychka et.al. 2018)

Obtaining uncertainties is facilitated by local conditional simulations



Simulate Gaussian
white noise over grid
(keep fixed)

Obtaining uncertainties is facilitated by local conditional simulations

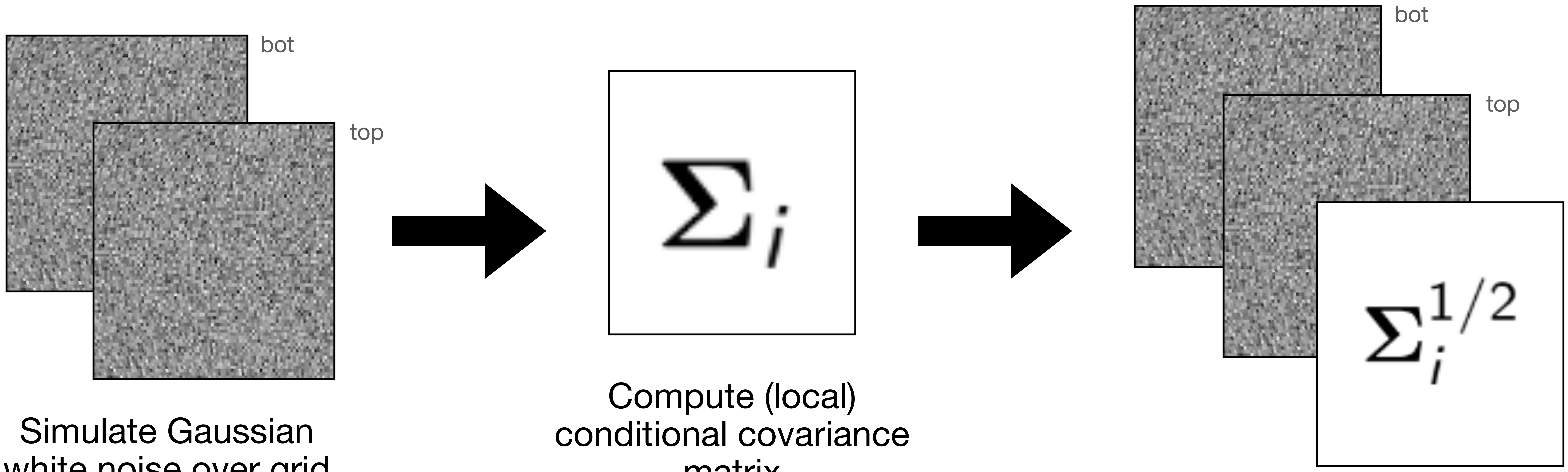


Simulate Gaussian white noise over grid (keep fixed)

$$\Sigma_i$$

Compute (local) conditional covariance matrix

Obtaining uncertainties is facilitated by local conditional simulations

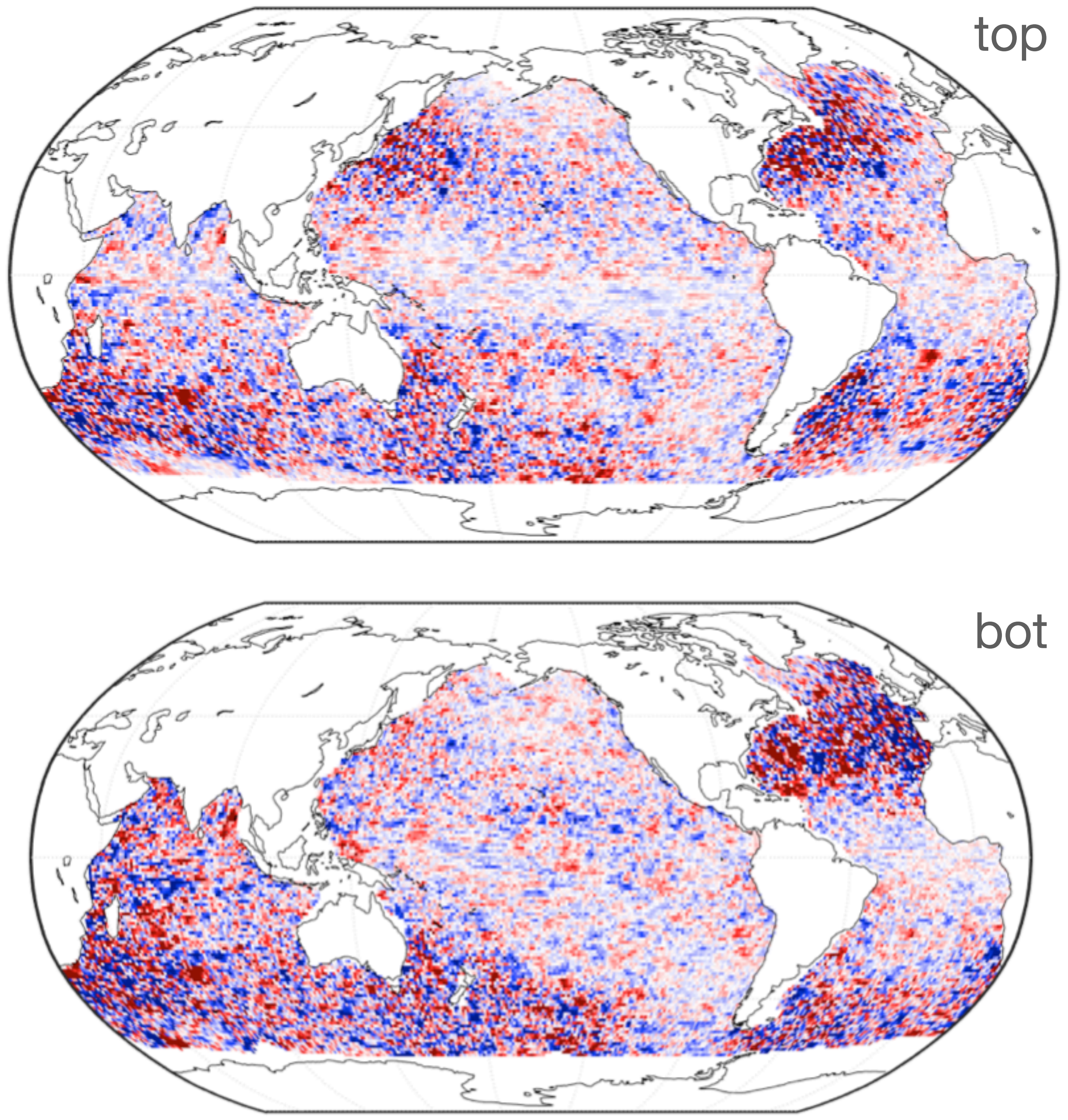


Simulate Gaussian white noise over grid (keep fixed)

Compute (local) conditional covariance matrix

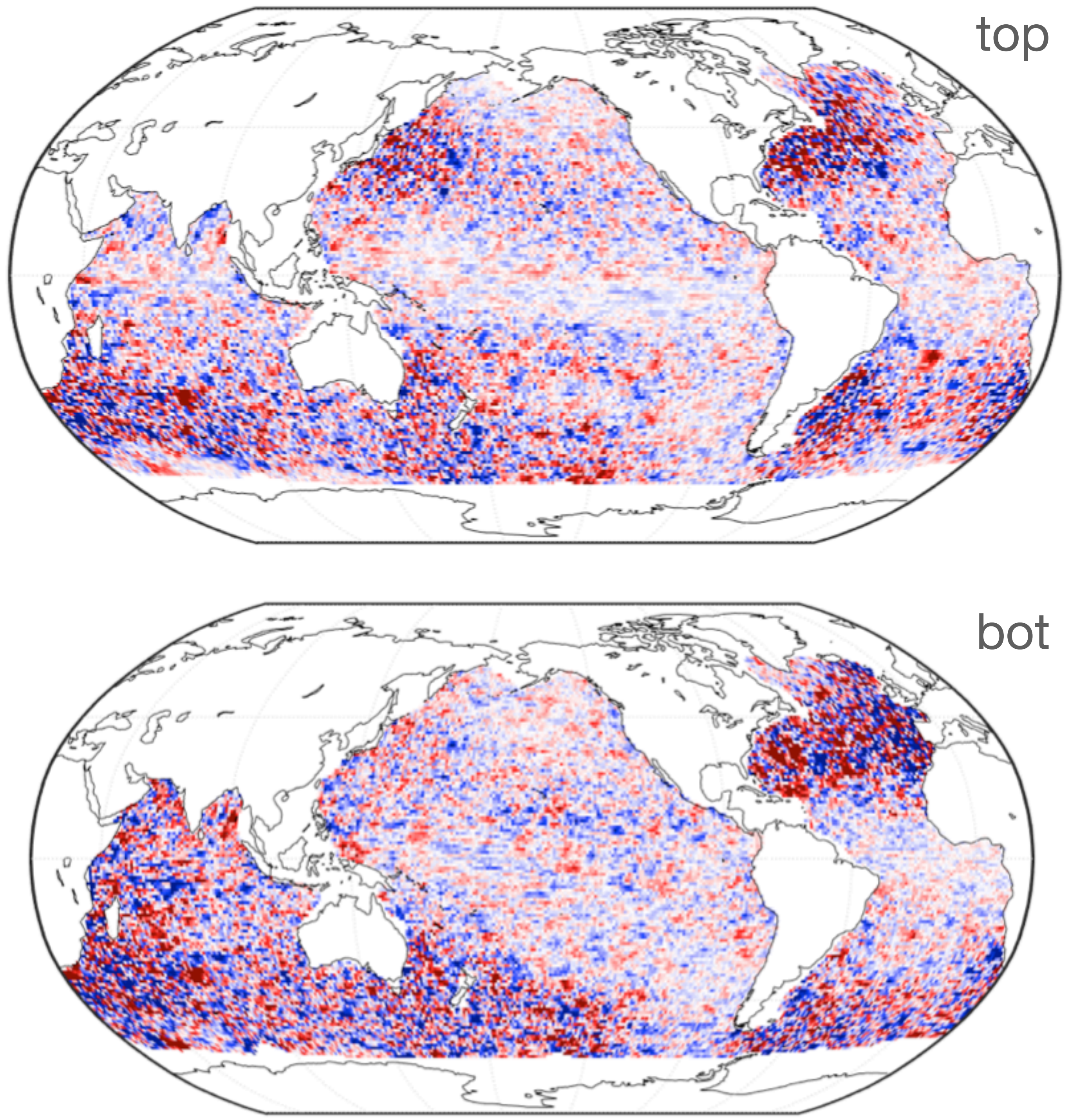
Take symmetric square root and multiply by white noise

Obtaining uncertainties is facilitated by local conditional simulations



Keep the center point
and repeat for all grid
points

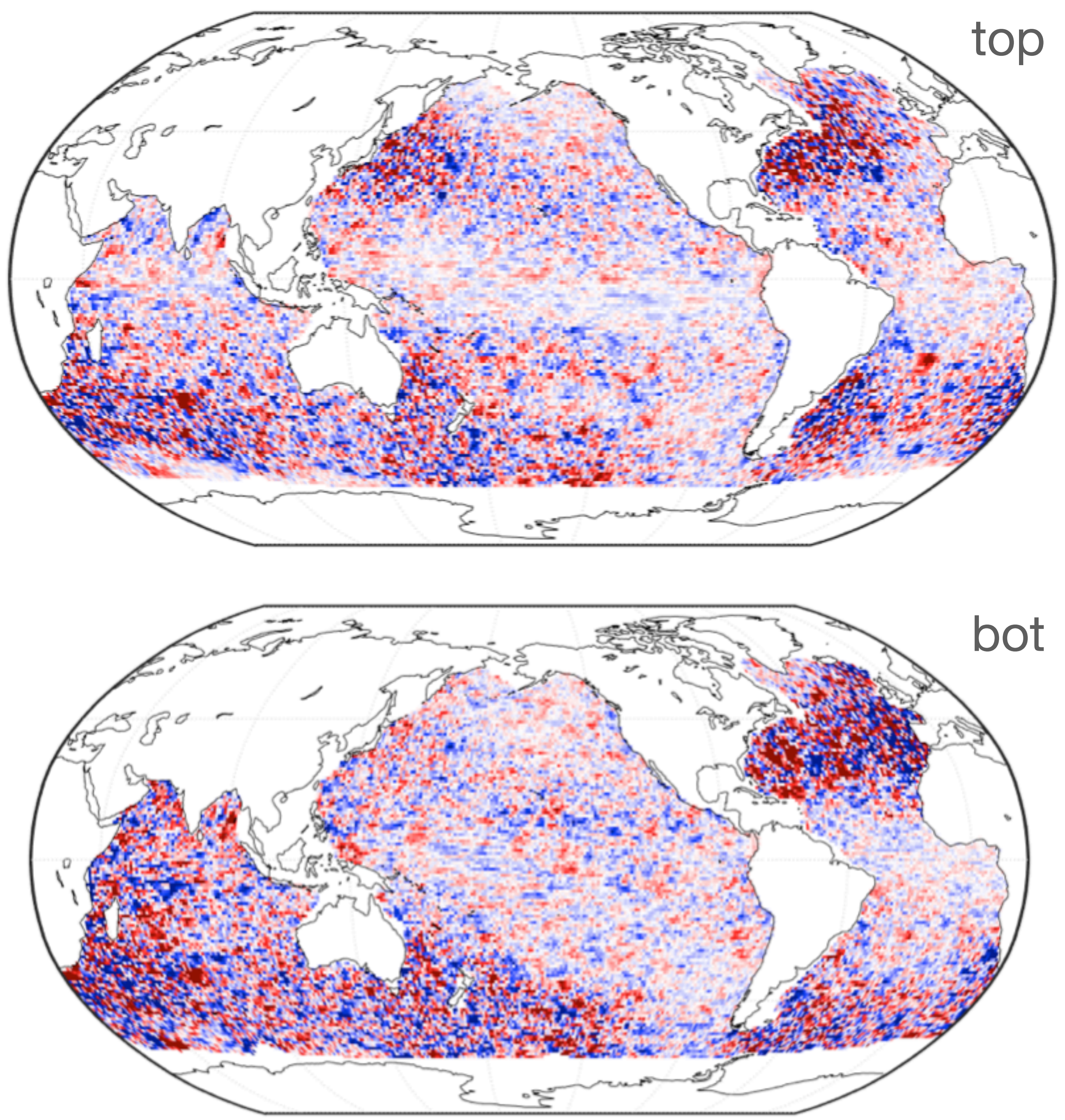
Obtaining uncertainties is facilitated by local conditional simulations



Keep the center point and repeat for all grid points

Repeat for desired number of ensemble members

Obtaining uncertainties is facilitated by local conditional simulations



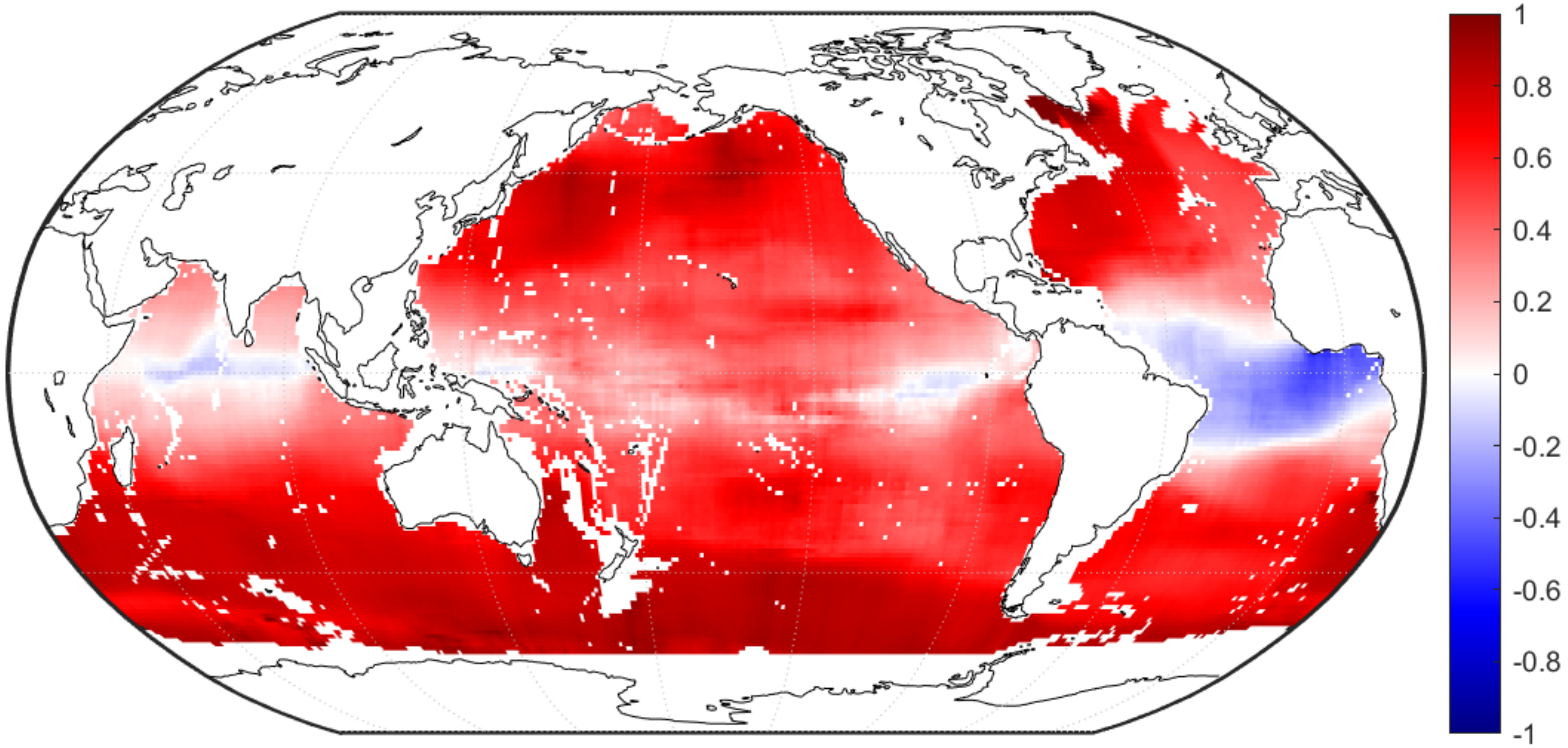
Keep the center point and repeat for all grid points

Repeat for desired number of ensemble members

We modeled the correlation, so sample variance of top + bottom layer integrated ensemble members is estimate of

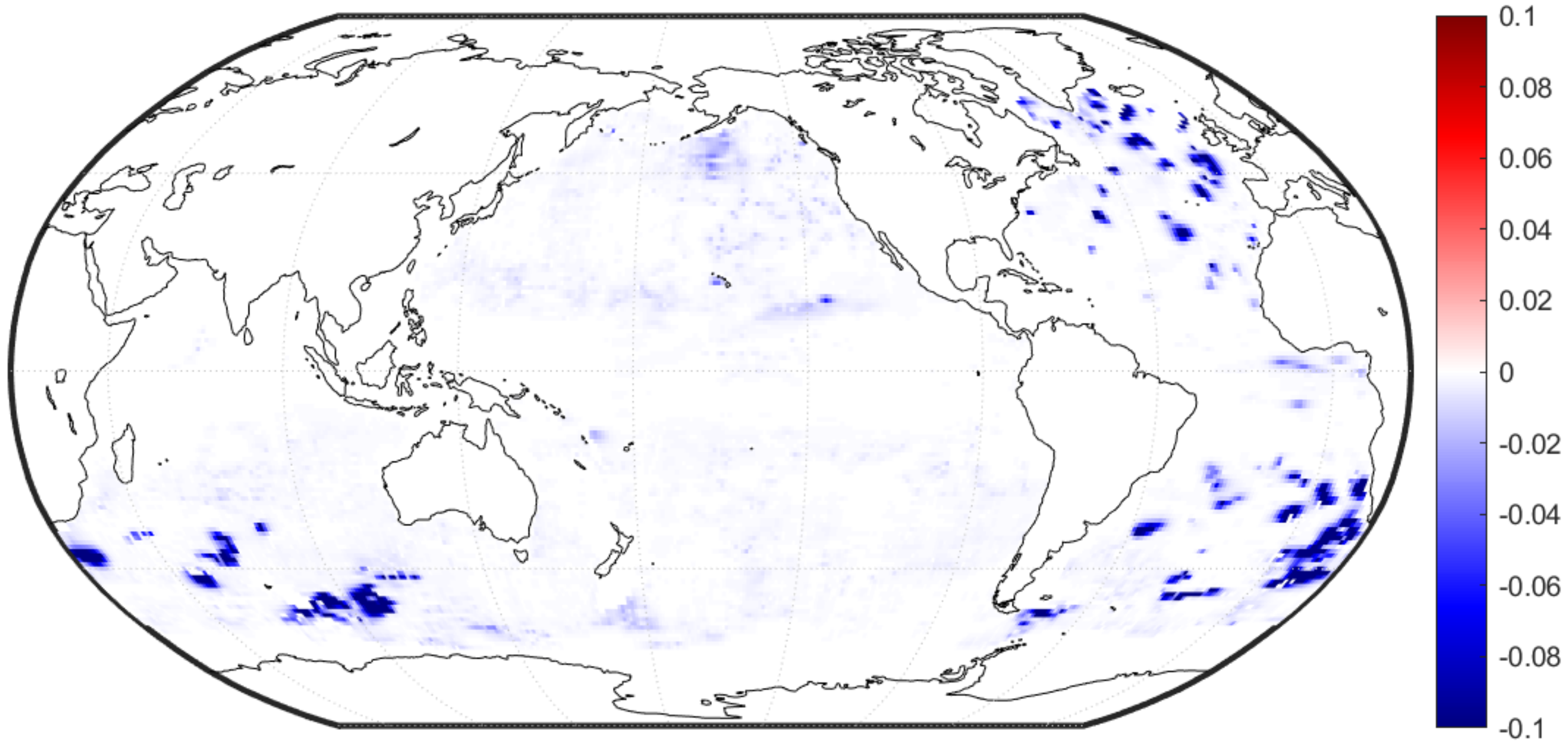
$$\mathrm{Var}(\mathrm{OHC}_{\mathrm{total}}|\mathrm{data})$$

Most ocean regions' temperature anomalies are positively correlated

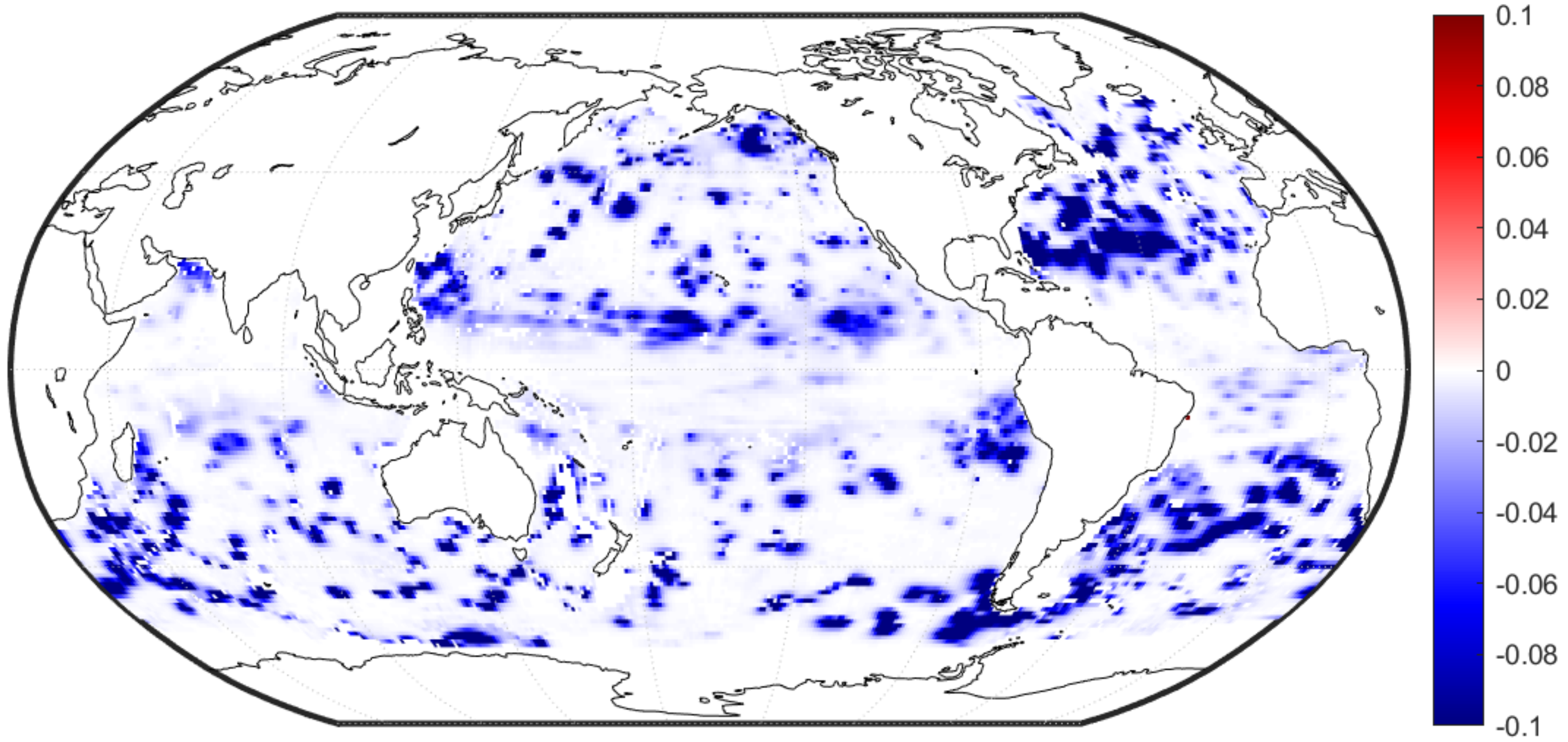


The bivariate model tends to produce lower kriging variances

$$\frac{\text{bivariate kriging variance - univariate kriging variance}}{\text{univariate kriging variance}} \quad (02/2010)$$



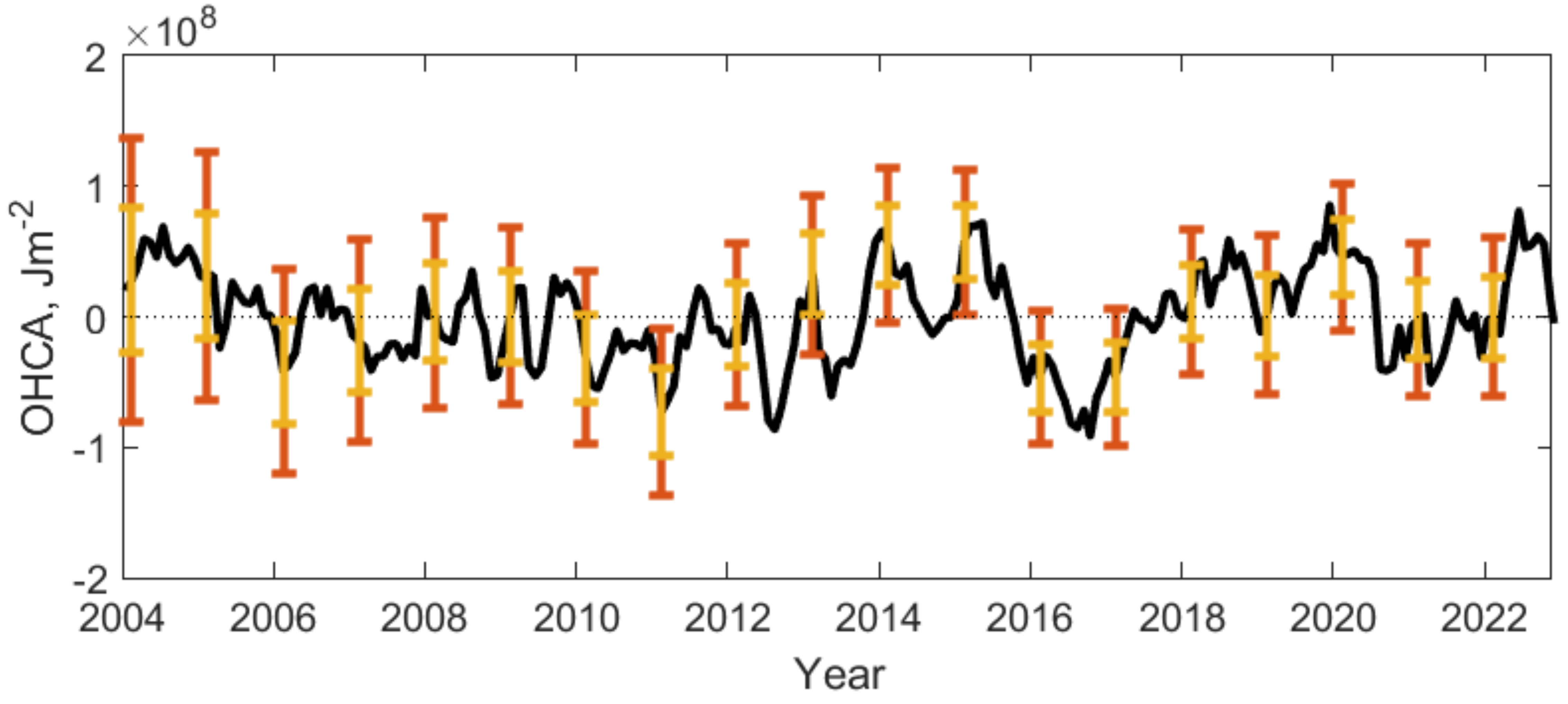
Top layer



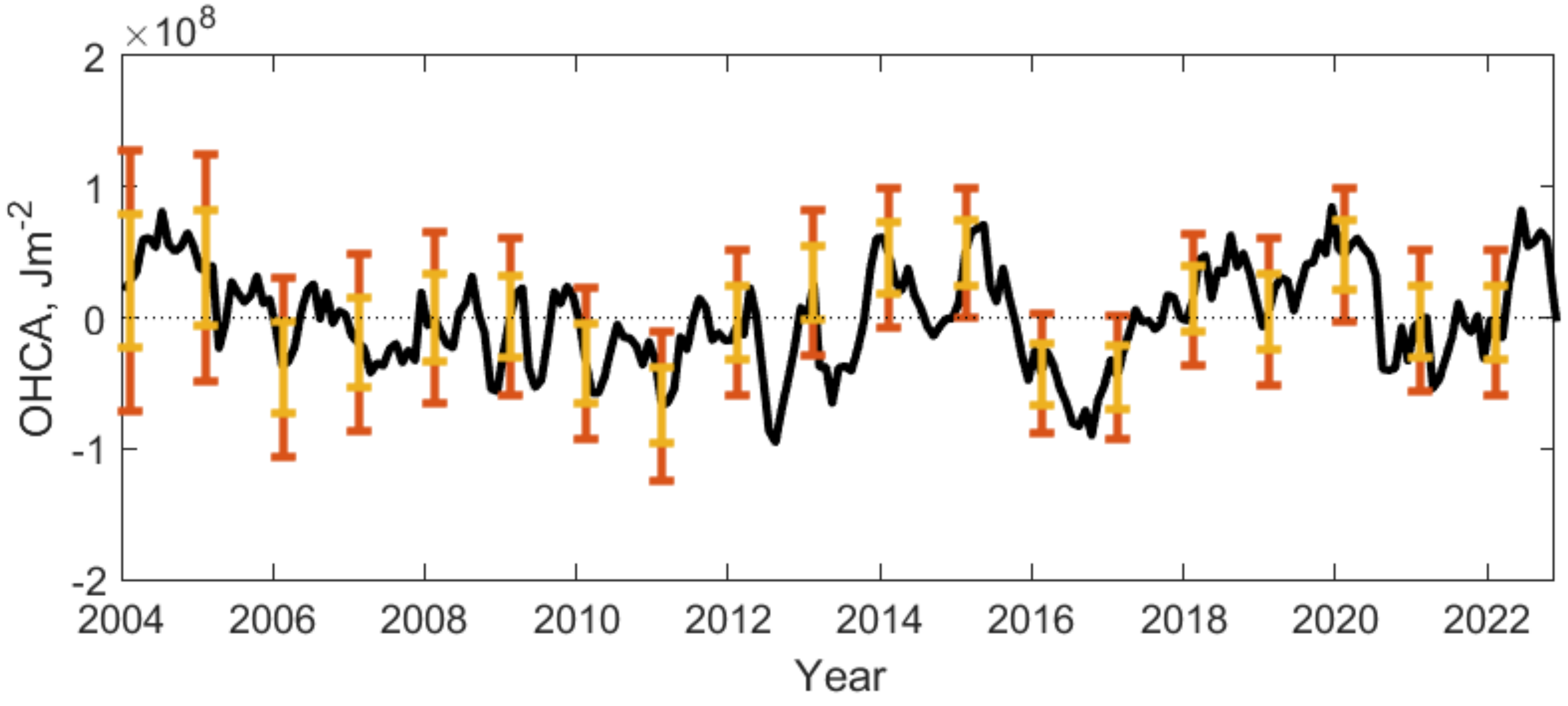
Bottom layer

Bivariate total (top + bottom) OHC uncertainties tend to be ~15% smaller than univariate

Global OHC anomaly estimates

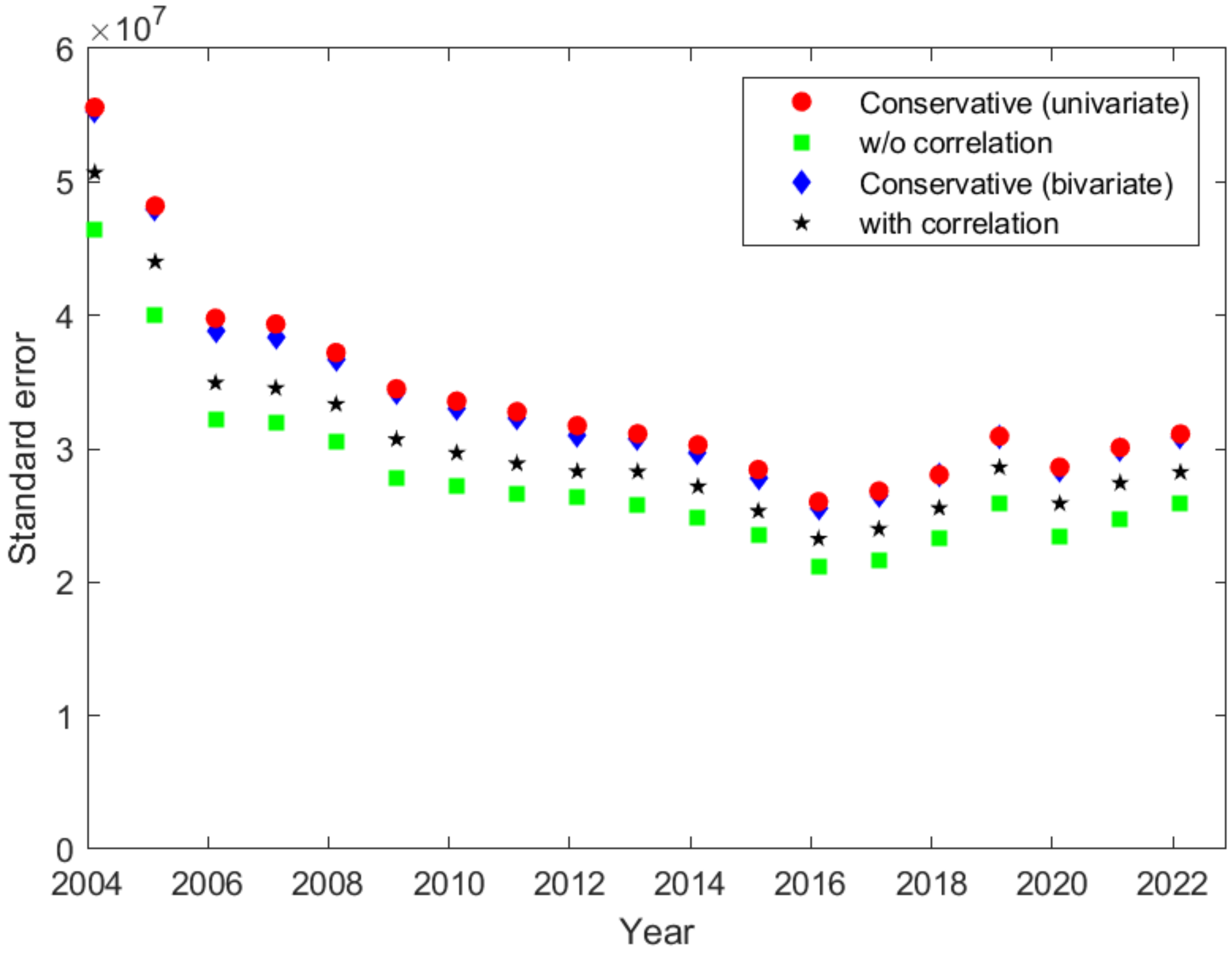


Univariate



Bivariate

Bivariate total (top + bottom) OHC uncertainties tend to be ~15% smaller than univariate



(w/o correlation)

$$\boxed{\sqrt{\mathrm{Var}(\mathrm{OHC}_{\mathrm{top}}|\mathrm{data}) + \mathrm{Var}(\mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})}}$$

(conservative)

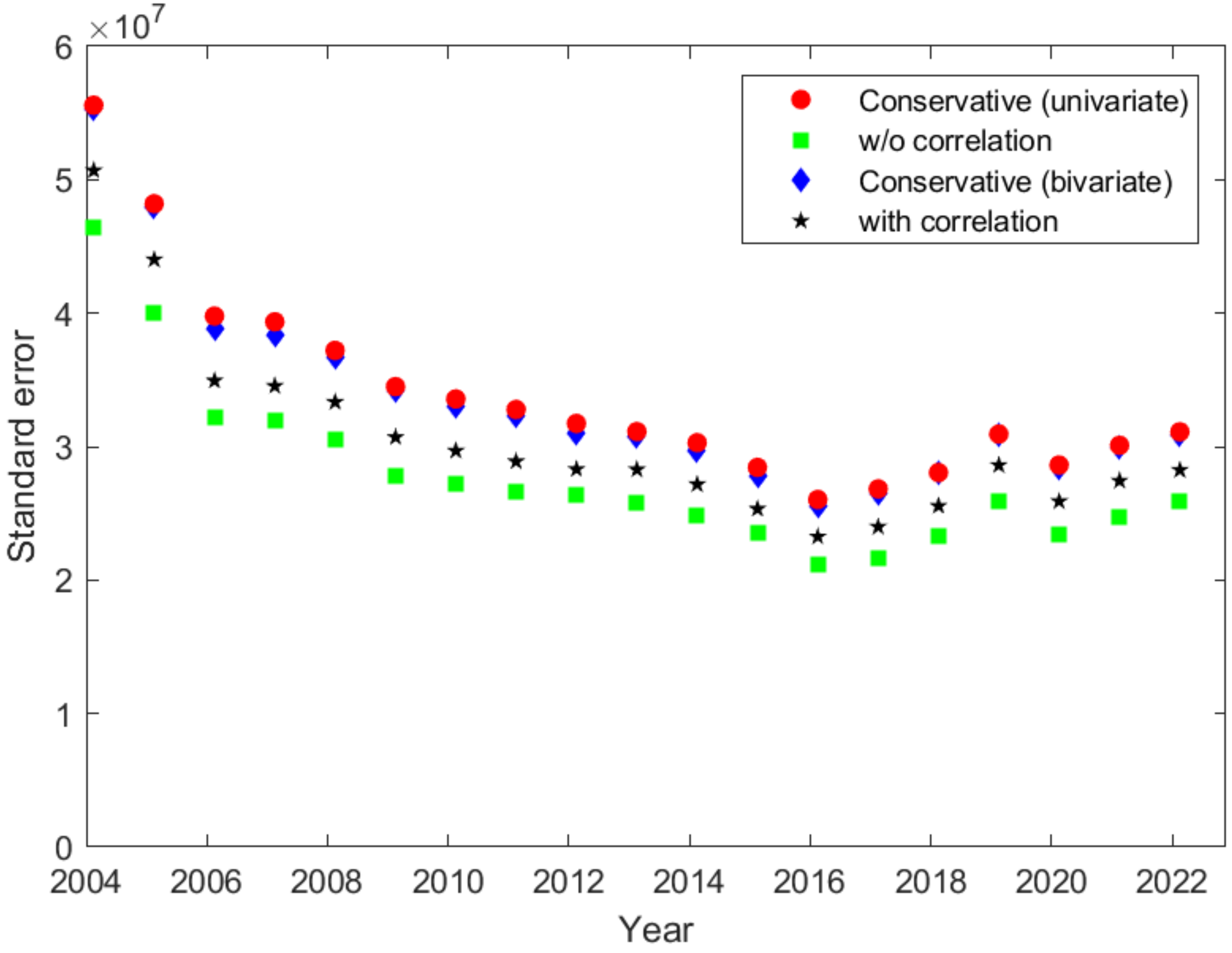
$$\boxed{\sqrt{\mathrm{Var}(\mathrm{OHC}_{\mathrm{top}}|\mathrm{data})} + \sqrt{\mathrm{Var}(\mathrm{OHC}_{\mathrm{bot}}|\mathrm{data})}}$$

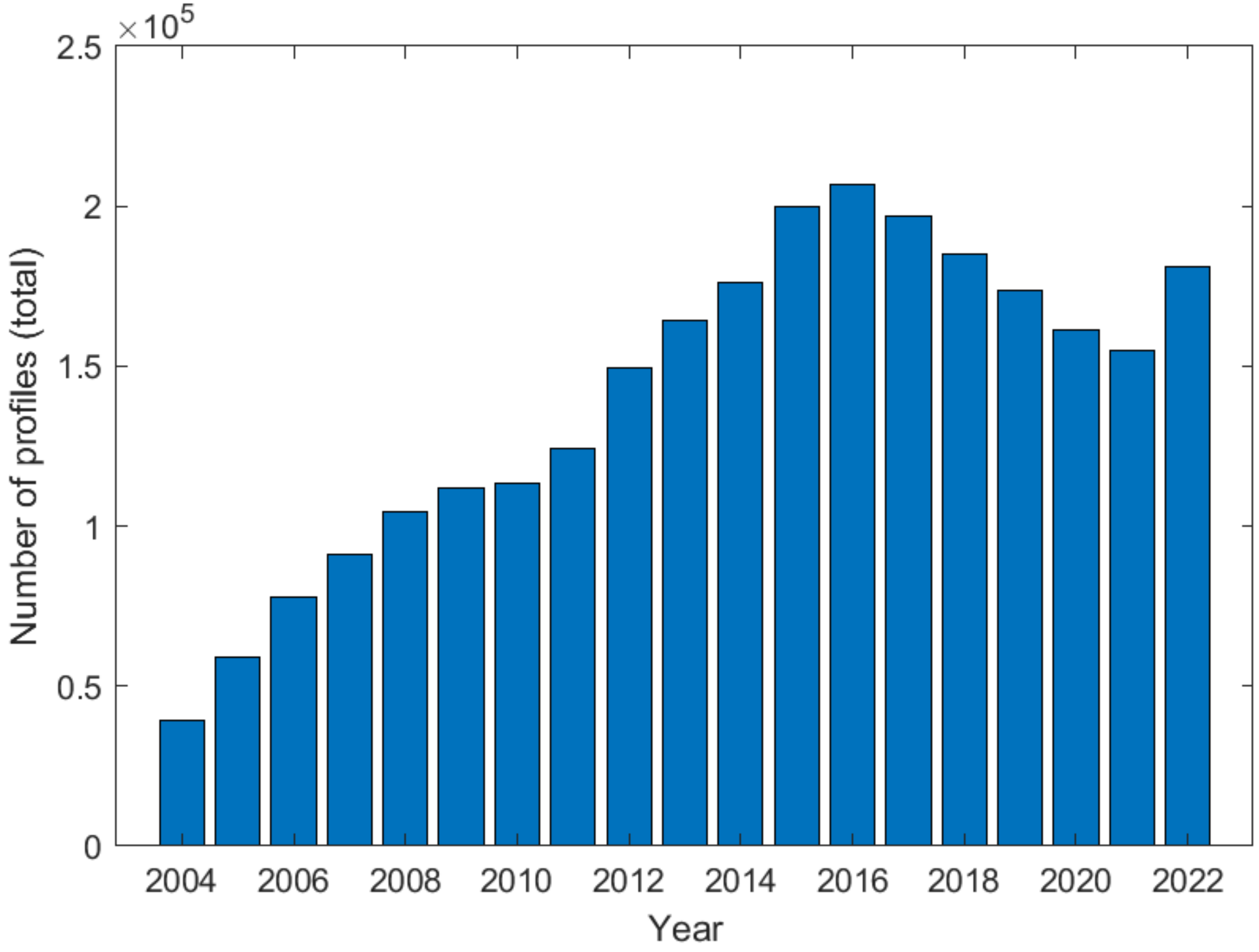
(with correlation)

$$\boxed{\sqrt{\mathrm{Var}(\mathrm{OHC}_{\mathrm{total}}|\mathrm{data})}}$$

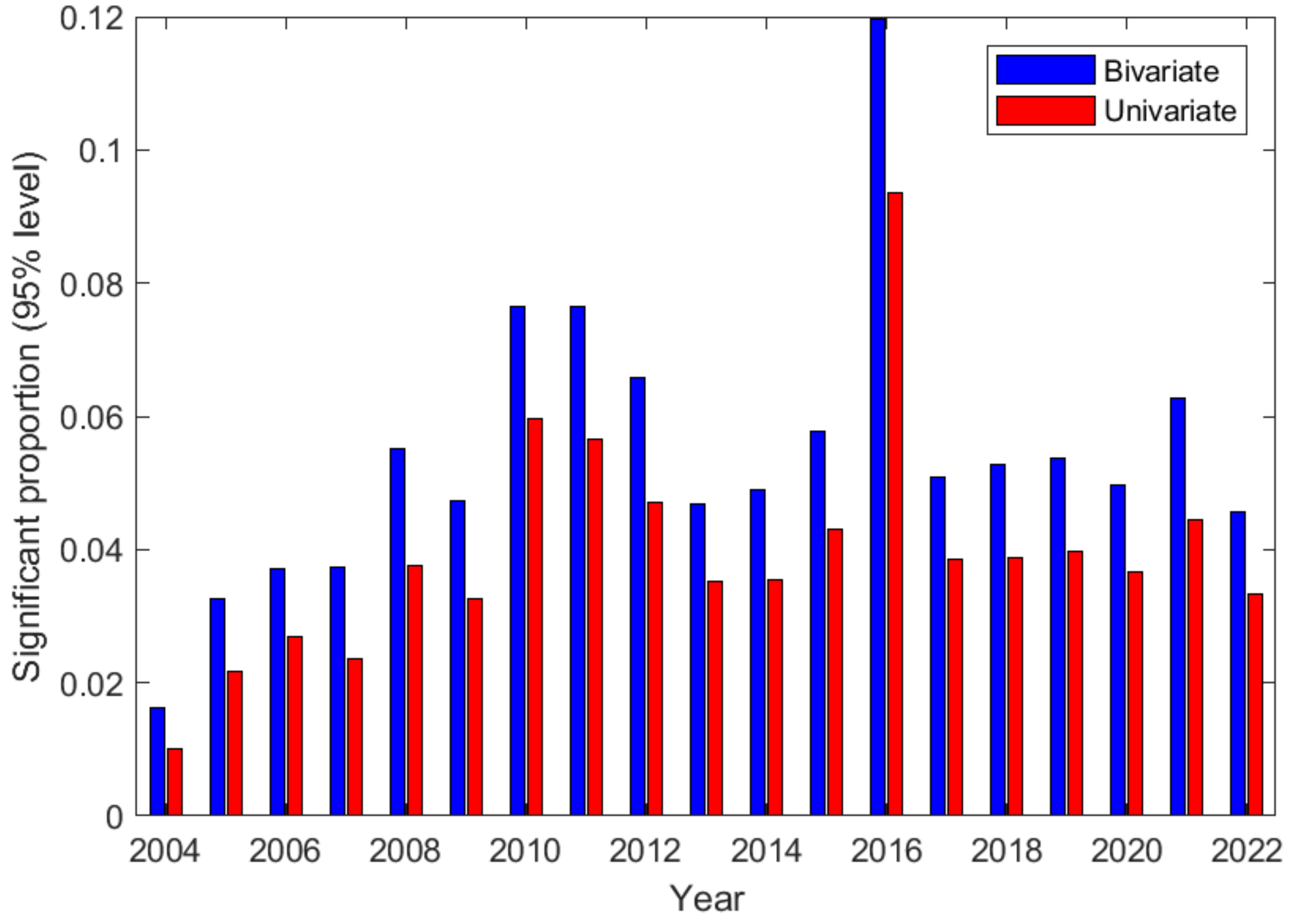
When we model the correlation, the uncertainties are **smaller** than the conservative and **larger** than those w/o the correlation.

The uncertainty trend follows the number of profiles/floats

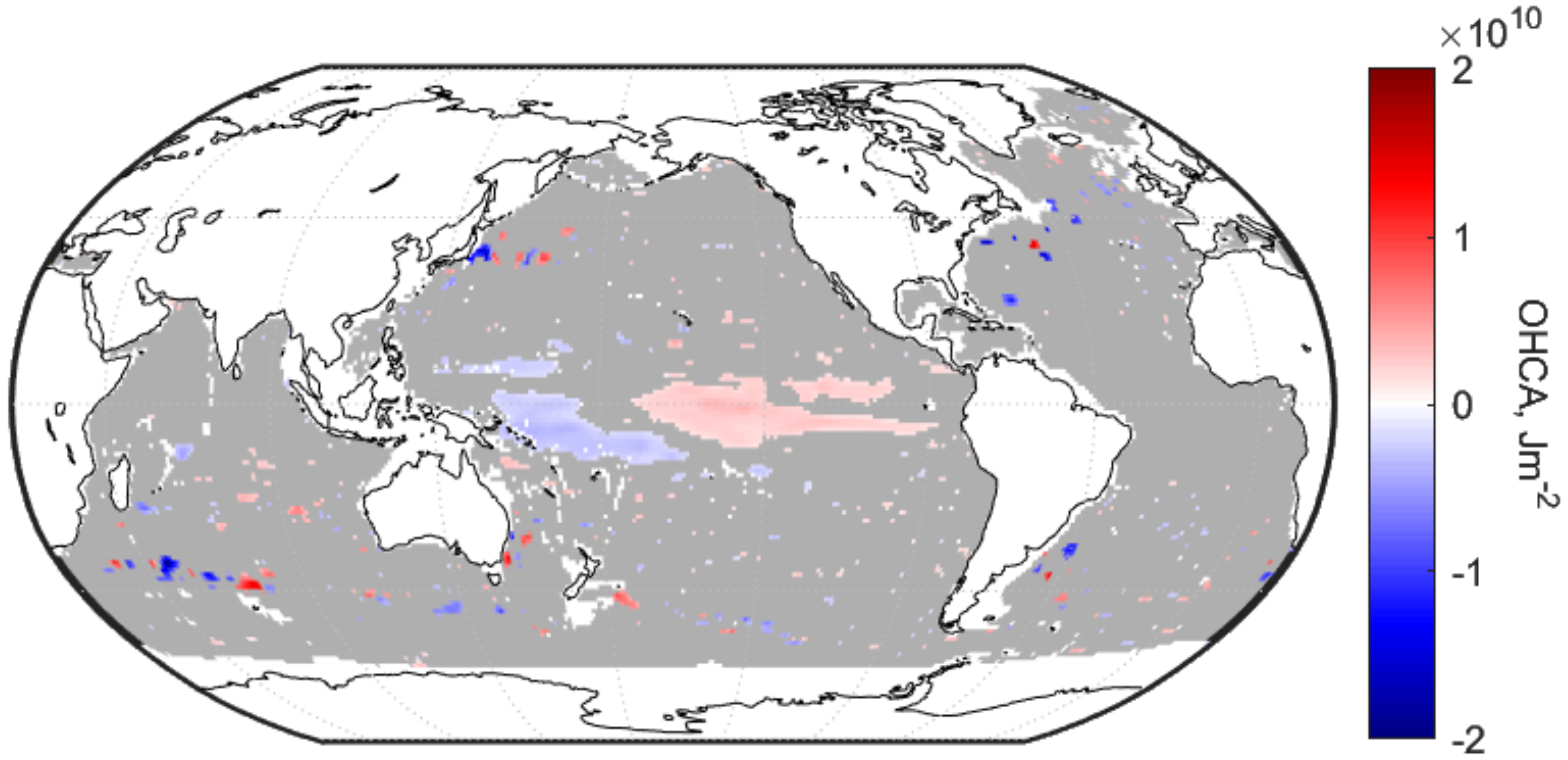




The equatorial OHC anomalies are consistently significant

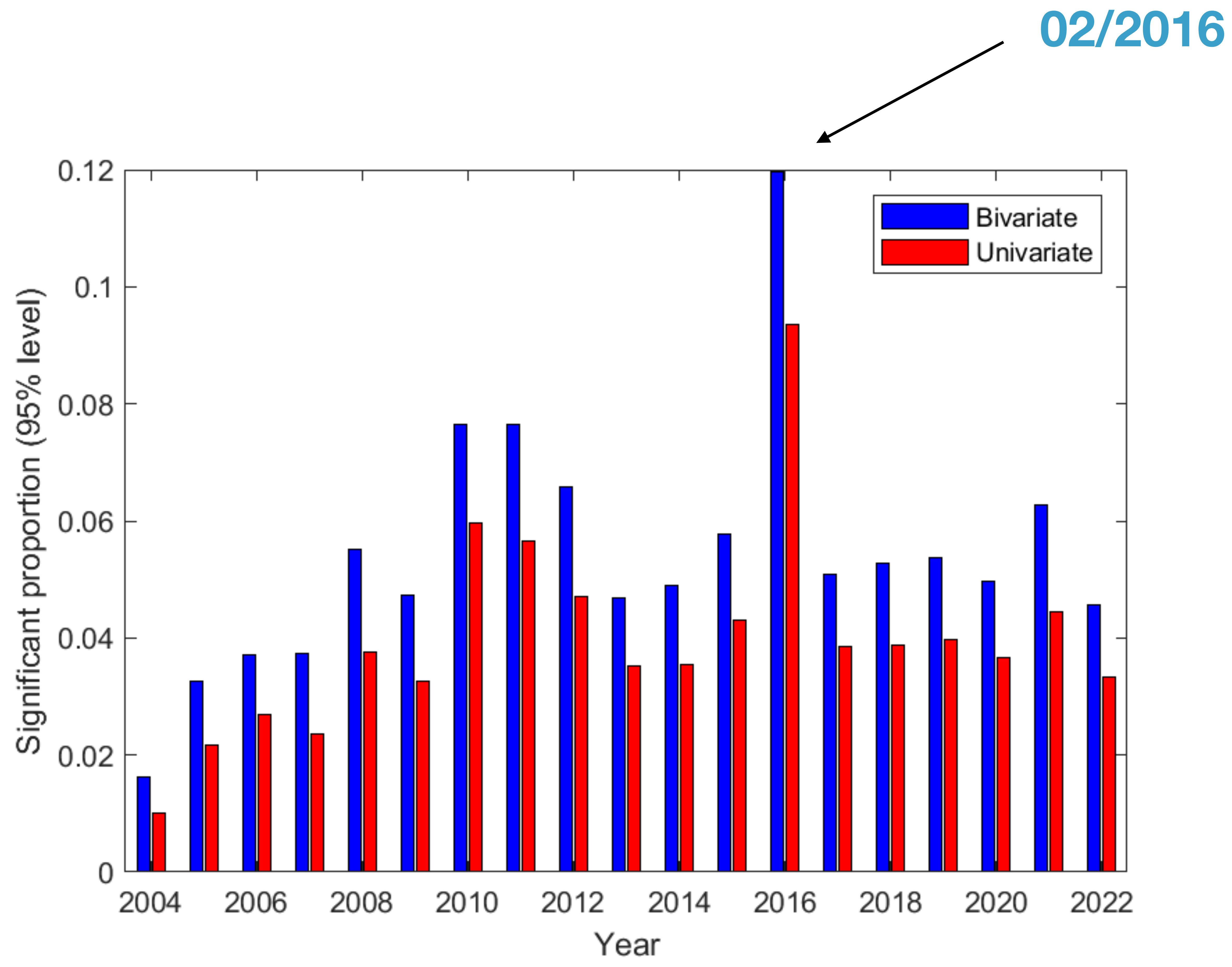


Proportion of grid points with significant anomalies over time (95% level)



Significant OHC anomalies (02/2010)

The equatorial OHC anomalies are consistently significant



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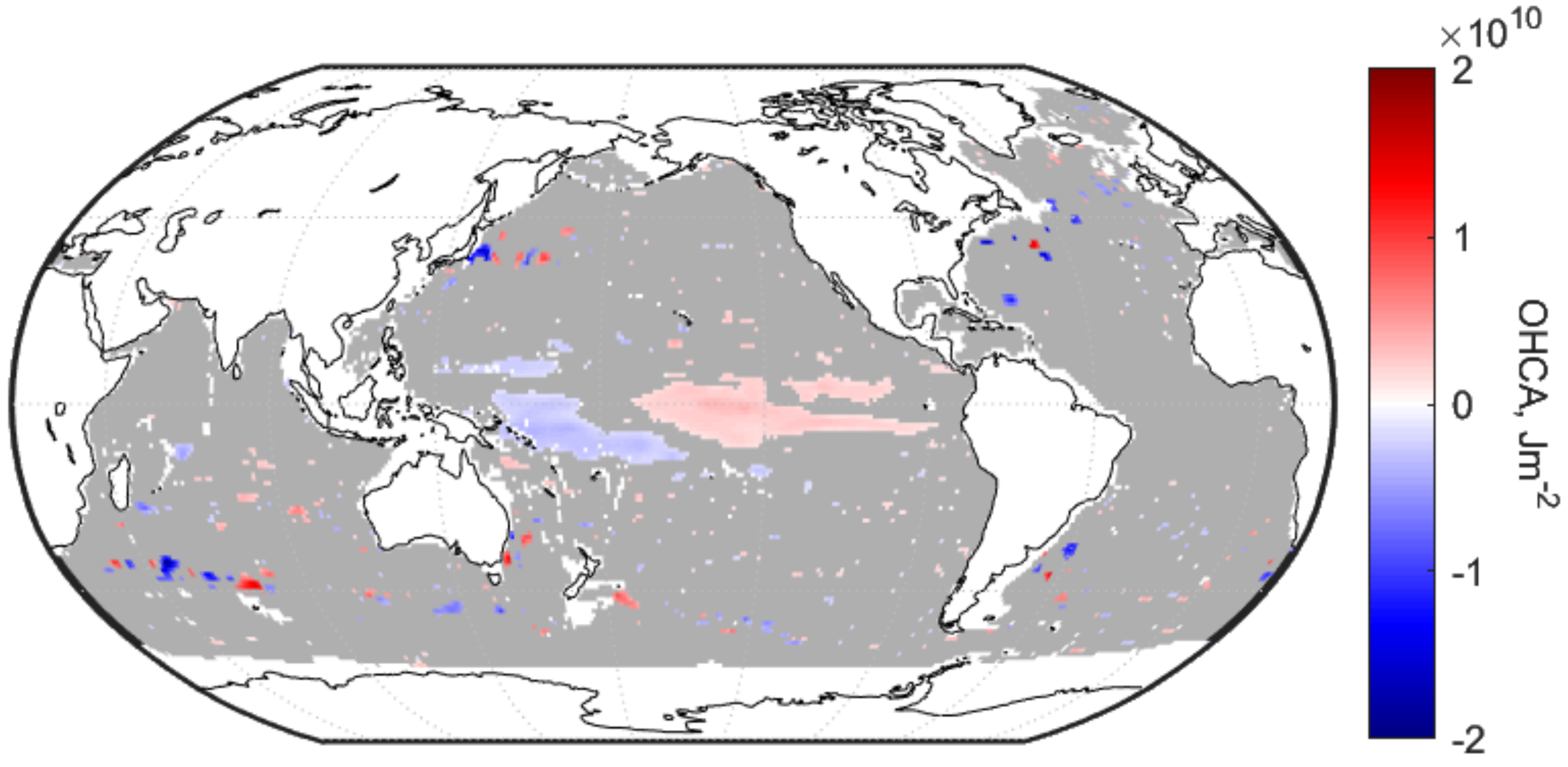
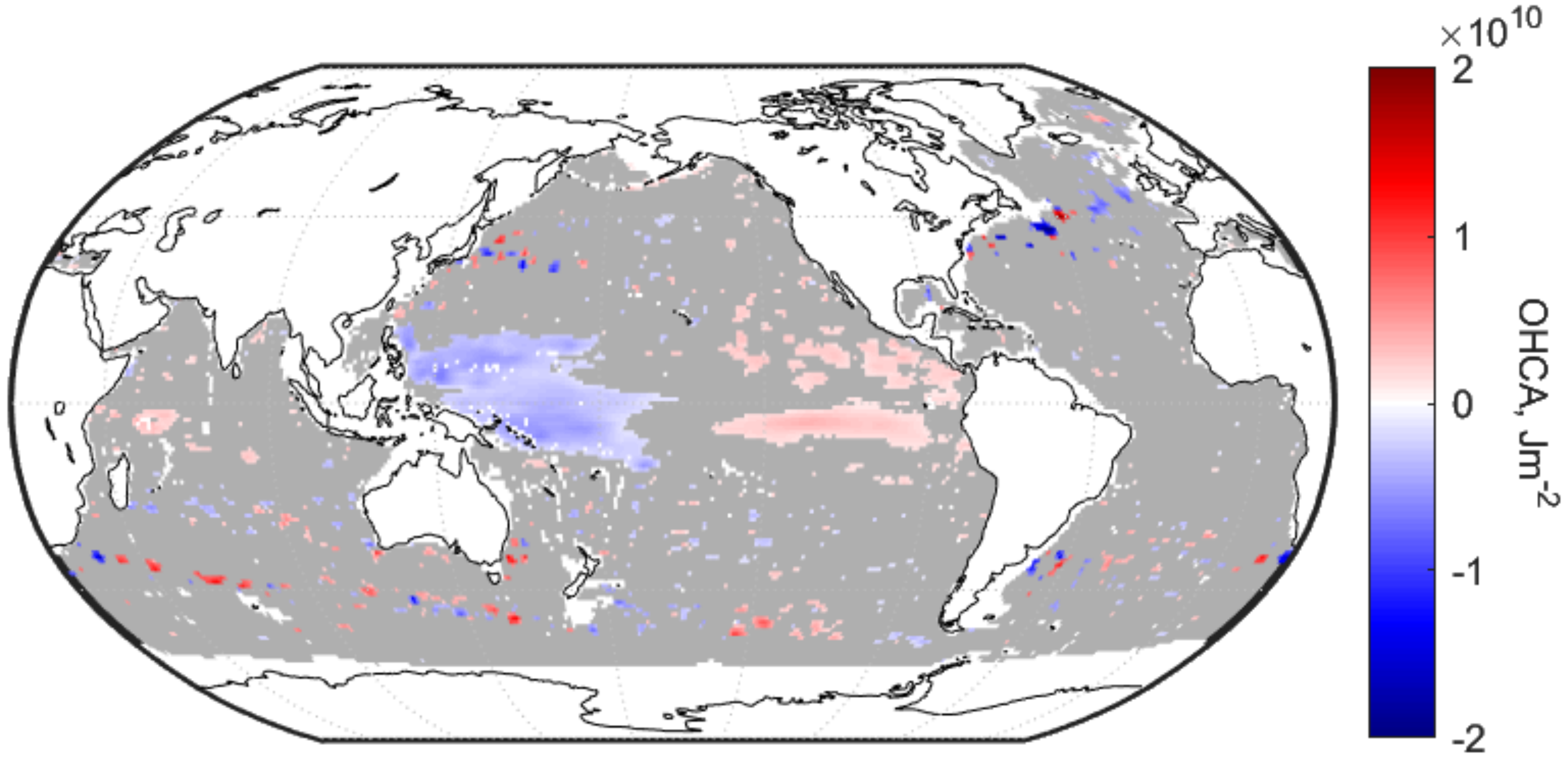
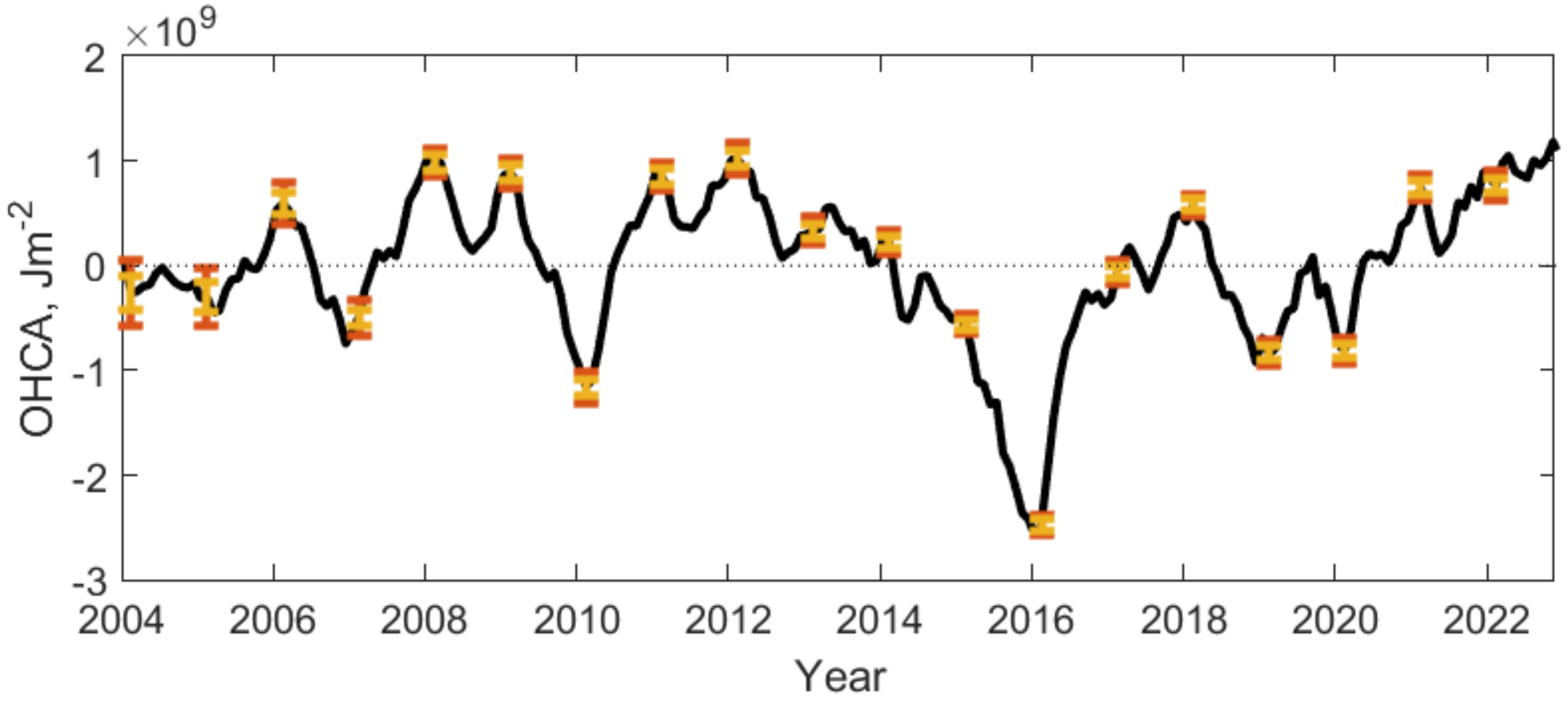


Significant OHC anomalies (02/2010)

The 2015-16 El Niño appears in the equatorial OHC anomalies

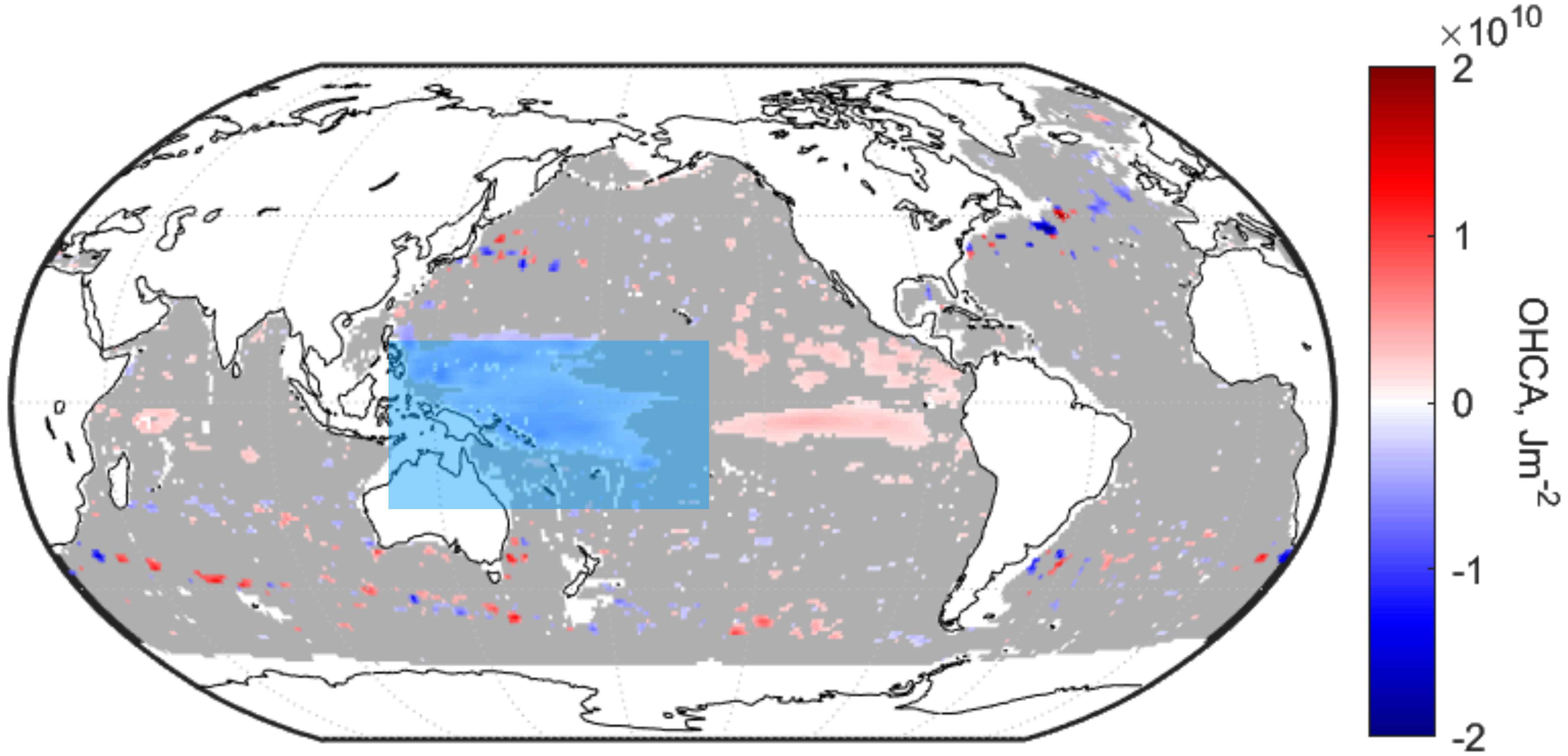


Significant OHC anomalies (02/2016)

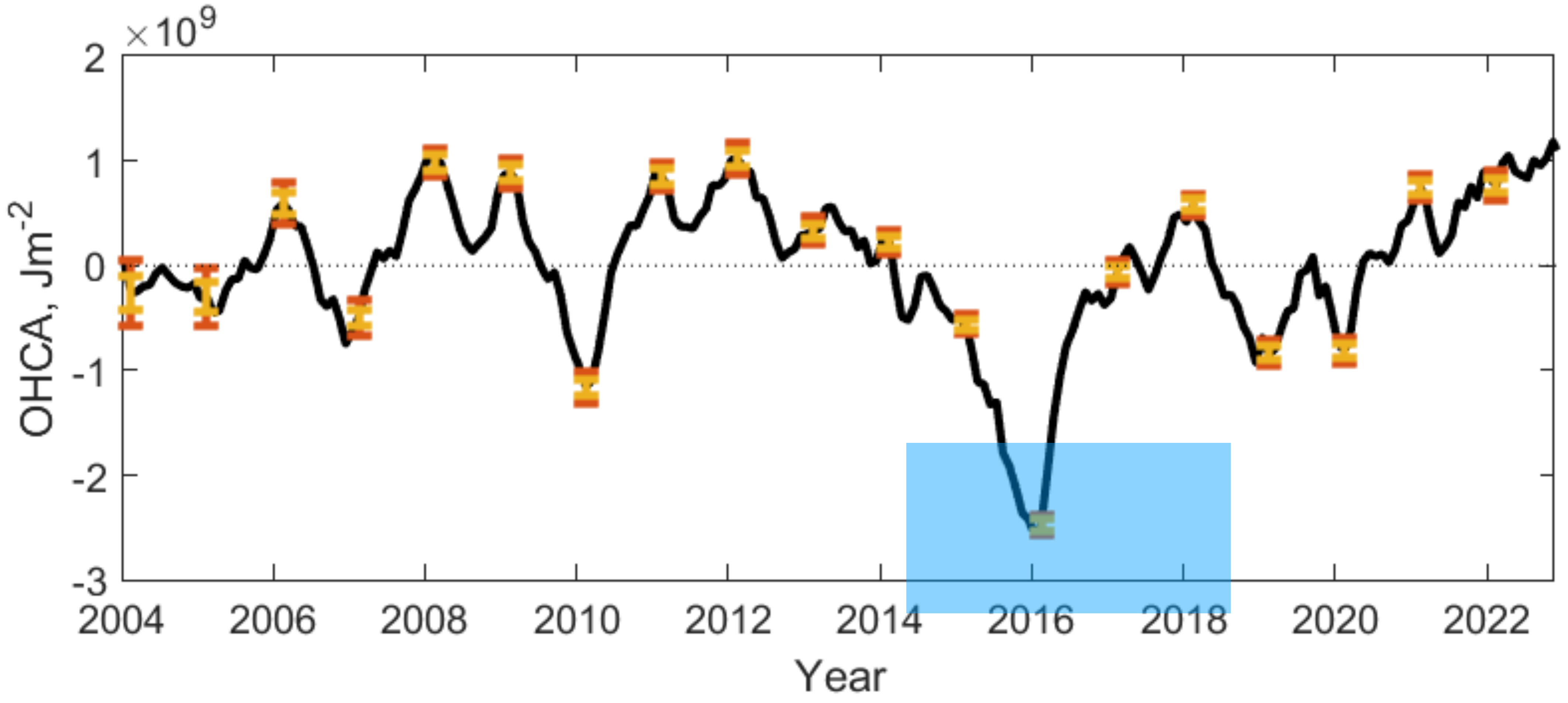


Western Equatorial Pacific OHC anomalies

The 2015-16 El Niño appears in the equatorial OHC anomalies



Significant OHC anomalies (02/2016)



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Future/related work

- Validate kriging variances and uncertainties (e.g. cross-validation)
- Uncertainties for mean field and climatological time trend
- Generalize GP regression model to more than two layers
- Apply model to other fields (e.g. SSH and OHC, oxygen and T/S)

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- Mapping synthetic profiles and comparing to model truth (ME4OH, poster by M. McCarthy / M. Palmer)
- Uncertainties for OHC trend, OHU, OHCA-ONI cross-correlation (talk by M. Kuusela)

Summary

- Estimating ocean heat content (with reliable uncertainties) is crucial for **tracking climate change**
- Due to having fewer observations deeper in the water column, we model the OHC in the top and bottom layers **separately**
- To model the uncertainties of the total OHC in the water column (top + bottom) we need to estimate the spatially-varying cross-layer **correlation**
- Empirically, using a bivariate GP model to estimate the correlation reduces the OHC anomaly uncertainties both for each layer separately and ~15% in the water column (top + bottom)

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Thank you!

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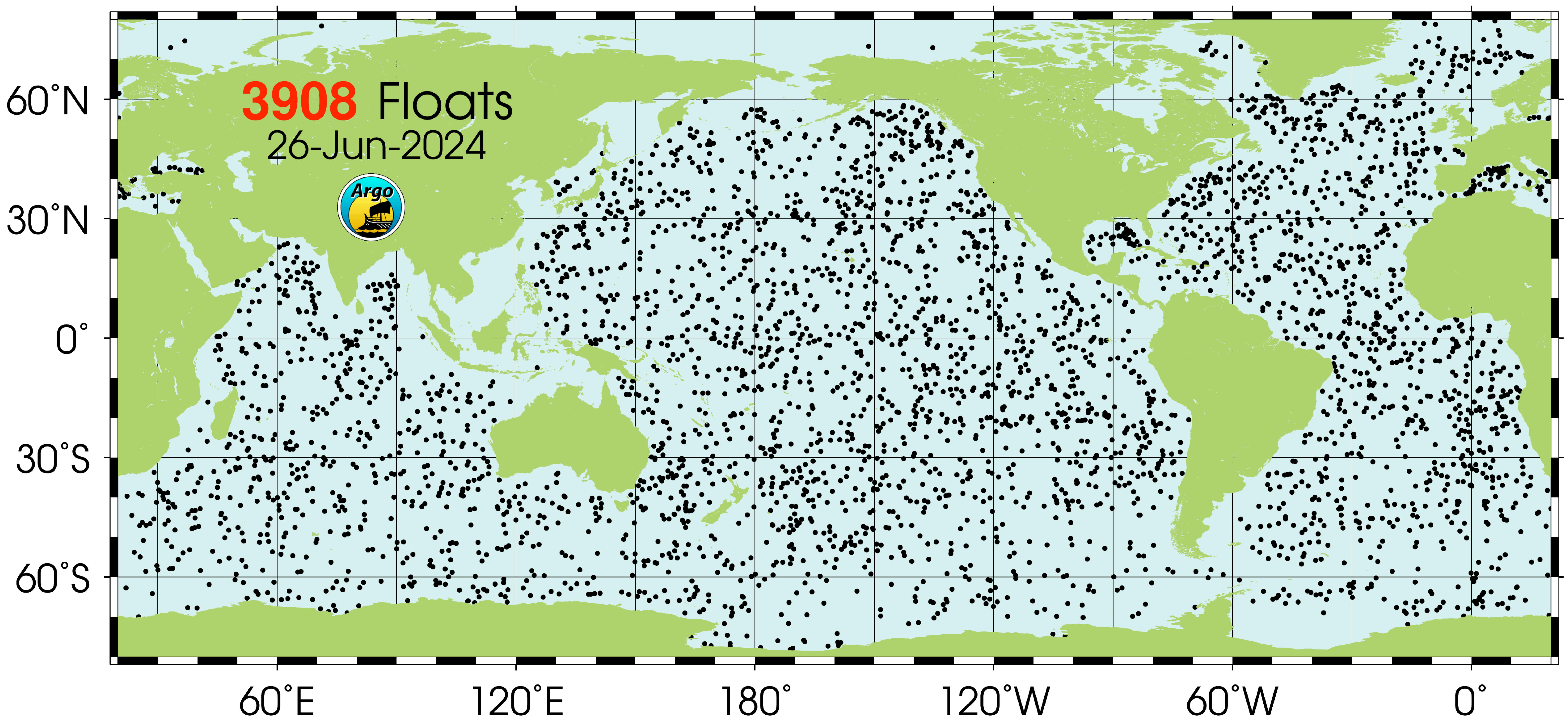
Backup

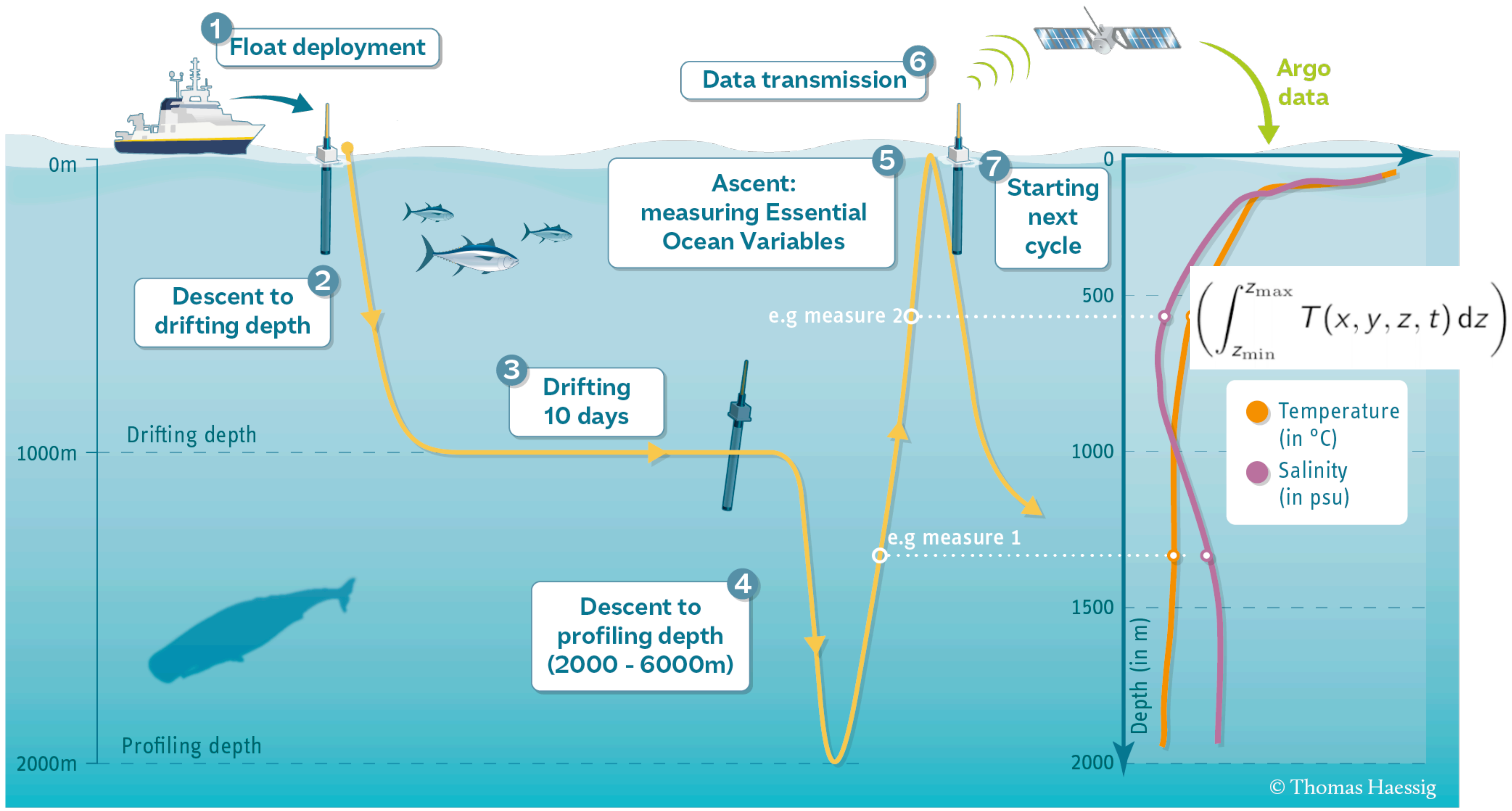
Argo floats are the state-of-the-art in ocean temperature measurements



(Argo Program)

OHC integral must be split into layers due to Argo data availability

$$\mathrm{OHC}(t) = \rho_0 c_{p,0} \int\int \left(\int_{z_{\min}}^{z_{\max}} T(x,y,z,t)\, \mathrm{d}z \right) \mathrm{d}x\, \mathrm{d}y$$



Top layer (15-975 dbar)

Bottom layer (975-1850 dbar)

The implementation is still computationally challenging

- For every grid point, use data in a 10 degree/3 month window
- Estimate GP model parameters numerically with MLE + BFGS algorithm
- How many grid points? **360 long x 180 lat = 64,800 grid points (!)**
- Embarrassingly parallel, but still computationally challenging
  - Fit parameters (optimize all): ~**80h @ PSC**
  - Obtain conditional simulations for Feb of every year: **~48h**